Problem Sheet – 2004
4^{th} – 5^{th} Grades

1. Lottery tickets are numbered from 000000 till 999999 (different tickets have different numbers). Let's define a ticket as "lucky" if the sum of all its 6 digits is an even number. Find the number of "unlucky" tickets.

A: 0 **B:** 100000 **C:** 499999 **D:** 500001 **N:** None of these

2. Find the least possible number of squares (their sides should be whole numbers) needed to make (without overlapping) a rectangle 6×7.

A: 4 **B:** 5 **C:** 6 **D:** 7 **N:** None of these

3. Decode the following sentence and find the missing letter:

X I M D U N I V U C S U U L M A P D U ? M I H I.

A: M **B:** O **C:** R **D:** U **N:** None of these

4. Square 3×3 is filled with 9 numbers (from 1 to 9) in some order. For each sub-square 2×2 the sum of all its four numbers is calculated. Let's define a *balance* of the square as the least of these four sums. Find the greatest possible value of the *balance*.

A: 23 **B:** 24 **C:** 25 **D:** 26 **N:** None of these

5. A patient is prescribed a special treatment. The treatment contains three distinct components, and the patient should take exactly one component per day starting from the beginning of the treatment. Find the greatest possible number of treatment days if for any three subsequent days during the treatment period the components taken during these three days (order matters!) are not the same as for any other three subsequent days.

A: 25 **B:** 27 **C:** 29 **D:** 31 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct statement? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. WE and HE represent two-digit numbers, so they cannot have 0 as their left digit.)

W E > H E

A: 144 **B:** 288 **C:** 324 **D:** 450 **N:** None of these

7. There are two empty bottles (for 13 L and 10 L) of irregular shape, and a water-pipe with a tap. Using only those objects, which exact volume of water is not possible to get in any of the bottles?

A: 1 L **B:** 2 L **C:** 3 L **D:** 4 L **N:** None of these

8. A 5-year period starts from Monday, January 1, and ends on Sunday, December 31. The subsequent 4-year period ends on Thursday. What day of the week does the next subsequent 3-year period end on?

A: Monday **B:** Wednesday **C:** Friday **D:** Sunday **N:** None of these

9. One brother is three times older than another one. In 10 years their father will be three times older than the older brother. In 22 years the father will be three times older than the younger brother. In how many years the total age of both brothers will be equal to their father's age? (Note: Age of a person (in years) is a whole number.)

A: 30 **B:** 36 **C:** 40 **D:** 44 **N:** None of these

10. Let's consider the following automaton. It accepts two cards (one after another) with numbers on them, and prints a new card with the number that equals to the first number doubled plus the second number tripled. It also returns original cards, and they could be used again. Starting from the two cards with number 4 on each, what number (on a card) could be obtained using the automaton described above?

A: 2000 **B:** 2004 **C:** 2008 **D:** 2012 **N:** None of these

Problem Sheet – 2004
6^{th} – 7^{th} Grades

1. Lottery tickets are numbered from 000000 till 999999 (different tickets have different numbers). Let's define a ticket as "lucky" if someone can obtain an even number using all its 6 digits (in any order) and addition / subtraction operations. It's not necessary to use both operations, but it's required to use each digit as many times as it appears in the ticket number. The usage of digits without an operation between them is prohibited. Find the number of "unlucky" tickets.

A: 0 **B:** 100000 **C:** 499999 **D:** 500001 **N:** None of these

2. Find the least possible number of squares (their sides should be whole numbers) needed to make (without overlapping) a rectangle 6×7.

A: 4 **B:** 5 **C:** 6 **D:** 7 **N:** None of these

3. Decode the following sentence and find the missing letter:

V A K B I L A S I Z Q I I J K U M B I ? K A F A.

A: D **B:** E **C:** H **D:** K **N:** None of these

4. Square 3×3 is filled with 9 numbers (from 1 to 9) in some order. For each sub-square 2×2 the sum of all its four numbers is calculated. Let's define a *balance* of the square as the greatest of these four sums. Find the least possible value of the *balance*.

A: 15 **B:** 16 **C:** 17 **D:** 18 **N:** None of these

5. A patient is prescribed a special treatment. The treatment contains three distinct components, and a patient should take exactly one component per day starting from the beginning of the treatment. Find the greatest possible number of treatment days if for any three subsequent days during the treatment period the components taken during these three days (order matters!) are not the same as for any other three subsequent days.

A: 25 **B:** 27 **C:** 29 **D:** 31 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct statement? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. CAT and RAT represent three-digit numbers, so they cannot have 0 as their left digit.)

C A T > R A T

A: 1008 **B:** 2004 **C:** 2016 **D:** 4032 **N:** None of these

7. There are two empty bottles (for 13 L and 10 L) of irregular shape, and a water-pipe with a tap. Using only those objects, which exact volume of water is not possible to get in any of the bottles?

A: 1 L **B:** 2 L **C:** 3 L **D:** 4 L **N:** None of these

8. A 5-year period starts from Monday, January 1, and ends on Sunday, December 31. The subsequent 4-year period ends on Thursday. What day of the week does the next subsequent 3-year period end on?

A: Monday **B:** Wednesday **C:** Friday **D:** Sunday **N:** None of these

9. Let's consider the following automaton. It holds 4 coins with weights (in ounces) *a*, *b*, *c*, and *d* (values are not available for a player). Two of them are regular (they have the same weight), and two others are false (their weights differ from each other and from the weight of a regular coin). The automaton keyboard contains only buttons a, b, c, d, +, –, *, /, (,), ?, and CLEAR. During an attempt the player can type any algebraic expression and press ? button in order to evaluate it. The automaton then displays the value of the expression or word ERROR, if the expression does not make sense (for instance, a + * b /, or a / (b – b)). What is the least possible number of attempts the player should make to figure out (for sure) at least one regular coin?

A: 1 **B:** 2 **C:** 3 **D:** 4 **N:** None of these

10. Let's consider the following automaton. It accepts two cards (one after another) with numbers on them, and prints a new card with the number that equals to the first number doubled plus the second number tripled. It also returns original cards, and they could be used again. Starting from the two cards with number 4 on each, what number (on a card) could be obtained using the automaton described above?

A: 2000 **B:** 2004 **C:** 2008 **D:** 2012 **N:** None of these

Problem Sheet – 2004
8th Grade

1. Lottery tickets are numbered from 000000 till 999999 (different tickets have different numbers). Let's define a ticket as "lucky" if either it has at least two equal digits, or someone can obtain a number divisible by 3 using all its 6 digits (in any order) and addition / subtraction operations. It's not necessary to use both operations, but it's required to use each digit as many times as it appears in the ticket number. The usage of digits without an operation between them is prohibited. Find the number of "unlucky" tickets.

A: 0 **B:** 333333 **C:** 333334 **D:** 500000 **N:** None of these

2. Find the least possible number of squares (their sides should be whole numbers) needed to make (without overlapping) a rectangle 9×10.

A: 5 **B:** 6 **C:** 8 **D:** 10 **N:** None of these

3. Decode the following sentence and find the missing number:

+ 4 + 18 – 16 – 14 + 16 + 10 + 6 – 24 – 18 – 18 + 10 + 6 – 14 + 2 – 26 – 16 – 8 + 10 – 18 + 12 – 24 – 18 – 14 + 16 + 18 – 16 – 22 – 18 – 24 + ? + 24 + 10 + 26.

A: 2 **B:** 4 **C:** 6 **D:** 8 **N:** None of these

4. Square 4×4 is filled with 16 numbers (from 1 to 16) in some order. For each sub-square 3×3 the sum of all its nine numbers is calculated. Let's define a *balance* of the square as the least of these four sums. Find the greatest possible value of the *balance*.

A: 92 **B:** 93 **C:** 94 **D:** 95 **N:** None of these

5. A patient is prescribed a special treatment. The treatment contains three distinct components, and a patient should take exactly one component per day starting from the beginning of the treatment. Find the greatest possible number of treatment days if for any three subsequent days during the treatment period the components taken during these three days (order matters!) are not the same as for any other three subsequent days.

A: 25 **B:** 27 **C:** 29 **D:** 31 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct statement? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. EUR and USD represent three-digit numbers, so they cannot have 0 as their left digit.)

E U R > U S D

A: 4800 **B:** 6048 **C:** 9120 **D:** 12096 **N:** None of these

7. Find the number of different solutions (in real numbers) for the following equation: $x = f(f(\ldots f(x) \ldots))$ (2004 times), where $f(x) = (3 \cdot x - 2) / (5 \cdot x - 3)$.

A: 0 **B:** 1 **C:** 2 **D:** 4 **N:** None of these

8. A bisector cuts a triangle into two isosceles triangles. Find the least possible value (in degrees) of an angle of the original triangle.

A: 20 **B:** 30 **C:** 36 **D:** 45 **N:** None of these

9. Let's consider the following automaton. It holds 10 coins with weights (in ounces) *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, and *j* (values are not available for a player). Two of them are regular (they have the same weight), and eight others are false (their weights differ from each other and from the weight of a regular coin). The automaton keyboard contains only buttons a, b, c, d, e, f, g, h, i, j, +, –, *, /, (,), ?, and CLEAR. During an attempt the player can type any algebraic expression and press ? button in order to evaluate it. The automaton then displays the value of the expression or word ERROR, if the expression does not make sense (for instance, a + * b /, or a / (b – b)). What is the least possible number of attempts the player should make to figure out (for sure) at least one regular coin?

A: 1 **B:** 2 **C:** 3 **D:** 4 **N:** None of these

10. Let's consider the following automaton. It accepts two cards (one after another) with numbers on them, and prints a new card with the number that equals to the first number doubled plus the second number tripled. It also returns original cards, and they could be used again. Starting from the two cards with number 4 on each, what number (on a card) could be obtained using the automaton described above?

A: 2000 **B:** 2004 **C:** 2008 **D:** 2012 **N:** None of these

Problem Sheet – 2004
9^{th} – 10^{th} Grades

1. Lottery tickets are numbered from 000000 till 999999 (different tickets have different numbers). Let's define a ticket as "lucky" if someone can obtain a number divisible by 4 using all its 6 digits (in any order) and addition / subtraction operations. It's not necessary to use both operations, but it's required to use each digit as many times as it appears in the ticket number. The usage of digits without an operation between them is prohibited. Find the number of "unlucky" tickets.

A: 0 **B:** 500000 **C:** 625000 **D:** 750000 **N:** None of these

2. Find the least possible number of squares (their sides should be whole numbers) needed to make (without overlapping) a rectangle 9×10.

A: 5 **B:** 6 **C:** 8 **D:** 10 **N:** None of these

3. Decode the following sentence and find the missing number:

+ 4 + 18 – 16 – 14 + 16 + 10 + 6 – 24 – 18 – 18 + 10 + 6 – 14 + 2 – 26 – 16 – 8 + 10 – 18 + 12 – 24 – 18 – 14 + 16 + 18 – 16 – 22 – 18 – 24 + ? + 24 + 10 + 26.

A: 2 **B:** 4 **C:** 6 **D:** 8 **N:** None of these

4. Square 4×4 is filled with 16 numbers (from 1 to 16) in some order. For each sub-square 3×3 the sum of all its nine numbers is calculated. Let's define a *balance* of the square as the greatest of these four sums. Find the least possible value of the *balance*.

A: 59 **B:** 60 **C:** 61 **D:** 62 **N:** None of these

5. A patient is prescribed a special treatment. The treatment contains three distinct components, and a patient should take exactly one component per day starting from the beginning of the treatment. Find the greatest possible number of treatment days if for any three subsequent days during the treatment period the components taken during these three days (order matters!) are not the same as for any other three subsequent days.

A: 25 **B:** 27 **C:** 29 **D:** 31 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct statement? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. DOG and CAT represent three-digit numbers, so they cannot have 0 as their left digit.)

D O G > C A T

A: 30240 **B:** 32000 **C:** 60480 **D:** 120960 **N:** None of these

7. Find the number of different solutions (in real numbers) for the following equation: $x = f(f(\ldots f(x) \ldots))$ (2004 times), where $f(x) = (x + 1/x) / 2$.

A: 0 **B:** 1 **C:** 2 **D:** 4 **N:** None of these

8. For any point M on the side AB of a non-isosceles triangle ABC let's define points K on the side AC and L on the side BC such as $MK \parallel BC$ and $ML \parallel AC$. Find for how many points M within AB corresponding triangle KLC is similar to the original triangle.

A: 1 **B:** 2 **C:** 3 **D:** 4 **N:** None of these

9. Let's consider the following automaton. It holds a trapezoid with subsequent sides a, b, c, and d (values are not available for a player). The automaton keyboard contains only buttons a, b, c, d, +, –, *, /, (,), ?, and CLEAR. During one attempt a player can type any algebraic expression and press ? button in order to evaluate it. The automaton then displays the value of the expression or word ERROR, if the expression does not make sense (for instance, a + * b /, or a / (b – b)). What is the least possible number of attempts the player should make to figure out (for sure) what sides (a and c or b and d) are the trapezoid's bases?

A: 1 **B:** 2 **C:** 3 **D:** 4 **N:** None of these

10. Which of the following numbers is a factor of $2\cdot(1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 + \ldots 497\cdot498\cdot499 + 498\cdot499\cdot500)$?

A: 2001 **B:** 2002 **C:** 2003 **D:** 2004 **N:** None of these

Problem Sheet – 2004
11^{th} – 12^{th} Grades

1. Lottery tickets are numbered from 000000 till 999999 (different tickets have different numbers). Let's define a ticket as "lucky" if someone can obtain a number divisible by 3 using all its 6 digits (in any order) and addition / subtraction operations. It's not necessary to use both operations, but it's required to use each digit as many times as it appears in the ticket number. The usage of digits without an operation between them is prohibited. Find the number of "unlucky" tickets.

A: 0 **B:** 33333 **C:** 34668 **D:** 36864 **N:** None of these

2. Find the least possible number of squares (their sides should be whole numbers) needed to make (without overlapping) a rectangle 9×10.

A: 5 **B:** 6 **C:** 8 **D:** 10 **N:** None of these

3. Find the missing number in the sequence: 1, 3, 3, 5, 3, 9, 3, ?, 5, 9, 3, 15, … .

A: 1 **B:** 3 **C:** 5 **D:** 7 **N:** None of these

4. Square 4×4 is filled with 16 numbers (from 1 to 16) in some order. For each sub-square 2×2 the sum of all its four numbers is calculated. Let's define a *balance* of the square as the least of these nine sums. Find the greatest possible value of the *balance*.

A: 32 **B:** 33 **C:** 34 **D:** 35 **N:** None of these

5. A patient is prescribed a special treatment. The treatment contains three distinct components, and a patient should take exactly one component per day starting from the beginning of the treatment. Find the greatest possible number of treatment days if for any three subsequent days during the treatment period the components taken during these three days (order matters!) are not the same as for any other three subsequent days.

A: 25 **B:** 27 **C:** 29 **D:** 31 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct statement? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. DOG, CAT, and RAT represent three-digit numbers, so they cannot have 0 as their left digit.)

D O G > C A T > R A T

A: 120000 **B:** 211680 **C:** 241920 **D:** 423360 **N:** None of these

7. Find the number of different solutions (in real numbers) for the following equation: $x = f(f(\ldots f(x) \ldots))$ (2004 times), where $f(x) = (x + 1/x) / 3$.

A: 0 **B:** 1 **C:** 2 **D:** 4 **N:** None of these

8. For any point M on the diameter AB of a semi-disk let's define points K and L on the semi-circle such as angles AMK and BML are equal. Let r be the radius (length) of the semi-disk, O – its center, d – the distance between M and O, and x – the common value of angles AMK and BML. The length of the chord KL depends only on the following:

A: r and x **B:** r and d **C:** d and x **D:** x **N:** None of these

9. Let's consider the following automaton. It holds a trapezoid with subsequent sides a, b, c, and d (values are not available for a player). The automaton keyboard contains only buttons a, b, c, d, +, –, *, /, (,), ?, and CLEAR. During one attempt a player can type any algebraic expression and press ? button in order to evaluate it. The automaton then displays the value of the expression or word ERROR, if the expression does not make sense (for instance, a + * b /, or a / (b – b)). What is the least possible number of attempts the player should make to figure out (for sure) what sides (a and c or b and d) are the trapezoid's bases?

A: 1 **B:** 2 **C:** 3 **D:** 4 **N:** None of these

10. Let's consider the following automaton. It accepts two cards (one after another) with numbers on them, and prints a new card with the number that equals to the first number doubled plus the second number tripled. It also returns original cards, and they could be used again. Starting from the two cards with number 1 on each, how many different numbers (on cards) less than 2004 (including original number 1) could be obtained using the automaton described above?

A: 328 **B:** 333 **C:** 336 **D:** 360 **N:** None of these

Answer Sheet – 2004

4^{th} – 5^{th} Grades

1. Ⓐ Ⓑ Ⓒ Ⓓ ●
2. Ⓐ ● Ⓒ Ⓓ Ⓝ
3. ● Ⓑ Ⓒ Ⓓ Ⓝ
4. Ⓐ ● Ⓒ Ⓓ Ⓝ
5. Ⓐ Ⓑ ● Ⓓ Ⓝ
6. Ⓐ ● Ⓒ Ⓓ Ⓝ
7. Ⓐ Ⓑ Ⓒ Ⓓ ●
8. Ⓐ Ⓑ Ⓒ ● Ⓝ
9. Ⓐ Ⓑ ● Ⓓ Ⓝ
10. Ⓐ Ⓑ Ⓒ Ⓓ ●

Total Score: 50

Answer Sheet – 2004

6^{th} – 7^{th} Grades

1. Ⓐ Ⓑ Ⓒ Ⓓ ●
2. Ⓐ ● Ⓒ Ⓓ Ⓝ
3. Ⓐ Ⓑ Ⓒ ● Ⓝ
4. Ⓐ ● Ⓒ Ⓓ Ⓝ
5. Ⓐ Ⓑ ● Ⓓ Ⓝ
6. Ⓐ Ⓑ ● Ⓓ Ⓝ
7. Ⓐ Ⓑ Ⓒ Ⓓ ●
8. Ⓐ Ⓑ Ⓒ ● Ⓝ
9. ● Ⓑ Ⓒ Ⓓ Ⓝ
10. Ⓐ Ⓑ Ⓒ Ⓓ ●

Total Score: 50

Answer Sheet – 2004

8th Grade

1. ● B C D N
2. A ● C D N
3. A ● C D N
4. A B ● D N
5. A B ● D N
6. A B C ● N
7. A B C D ●
8. A B ● D N
9. A ● C D N
10. A B C D ●

Total Score: 50

Answer Sheet – 2004

9^{th} – 10^{th} Grades

1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	●
2.	Ⓐ	●	Ⓒ	Ⓓ	Ⓝ
3.	Ⓐ	●	Ⓒ	Ⓓ	Ⓝ
4.	●	Ⓑ	Ⓒ	Ⓓ	Ⓝ
5.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ
6.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ
7.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ
8.	Ⓐ	●	Ⓒ	Ⓓ	Ⓝ
9.	●	Ⓑ	Ⓒ	Ⓓ	Ⓝ
10.	Ⓐ	Ⓑ	Ⓒ	●	Ⓝ

Total Score: 50

Answer Sheet – 2004

11^{th} – 12^{th} Grades

1.	Ⓐ	Ⓑ	Ⓒ	●	Ⓝ
2.	Ⓐ	●	Ⓒ	Ⓓ	Ⓝ
3.	Ⓐ	Ⓑ	Ⓒ	●	Ⓝ
4.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ
5.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	●
7.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ
8.	●	Ⓑ	Ⓒ	Ⓓ	Ⓝ
9.	●	Ⓑ	Ⓒ	Ⓓ	Ⓝ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	●

Total Score: 50