Problem Sheet – 2003

4^{th} – 5^{th} Grades

1. Lottery tickets are numbered from 000000 till 999999 (different tickets have different numbers). Let's define a ticket as "lucky", if someone can obtain 0 using all its digits (in any order), parentheses, and addition / subtraction / multiplication operations. It's not necessary to use all operations or parentheses, but it's required to use each digit as many times as it appears in the ticket number. The usage of digits without any operation or parenthesis between them is prohibited. Find the number of "unlucky" tickets.

A: 0 **B:** 1000 **C:** 100000 **D:** 500000 **N:** None of these

2. Square 3×3 filled in with 9 integers (all these 9 numbers are different). Let's multiply all 3 numbers within each row and each column, and obtain 6 products (some of them could be equal). Find the greatest possible number of equal products. For instance, if all 6 products could be equal to each other, the answer is 6, if only 5 products could be equal to each other, the answer is 5, and so on.

A: 2 **B:** 4 **C:** 5 **D:** 6 **N:** None of these

3. Find the number of all ways rectangle 2×10 could be cut into 10 dominoes (rectangles 2×1 or 1×2).

A: 64 **B:** 81 **C:** 89 **D:** 98 **N:** None of these

4. Let's define a one-way rook as a rook that can move / attack only in one direction (left, right, forward, or backward). In other words, a one-way rook is a rook with "face" that can move / attack only in the direction its "face" looks. Find the greatest possible number of one-way rooks that could be arranged on a standard 8×8 chessboard without attacking each other.

A: 24 **B:** 28 **C:** 30 **D:** 32 **N:** None of these

5. Find the number of all 11-letter "words" (it is not necessary that they make any sense in English) that satisfy the following conditions: only capital letters are used, the first letter is E, the last one is K, and any two neighboring letters are also neighbors in the alphabet (for instance, letters T and S).

A: 41 **B:** 43 **C:** 45 **D:** 50 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct addition example? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. NINE and SIX represent four-digit and three-digit numbers, so they cannot have 0 as their left digit.)

```
  N I N E
+   S I X
---------
  2 0 0 3
```

A: 0 **B:** 2 **C:** 4 **D:** 6 **N:** None of these

7. Find the missing number in the sequence: 31, ?, 90, 120, 151, 181, ...

A: 56 **B:** 60 **C:** 61 **D:** 62 **N:** None of these

8. A "black box" contains 10 blue, 20 red, 30 green, and 40 white chips that are the same (except color). Find the least possible number of chips that should be randomly taken out of the "black box" to guarantee that at least 3 different colors obtained.

A: 60 **B:** 70 **C:** 80 **D:** 81 **N:** None of these

9. One brother is twice older than another one. In 20 years their father will be twice older than the older brother. In 40 years the father will be twice older than the younger brother. What was the total age of these three people 5 years ago? (Note: Age of a person (in years) is a whole number.)

A: 65 **B:** 68 **C:** 75 **D:** 78 **N:** None of these

10. All pages in a book are numbered in a standard way starting from the number 1. It appeared that some digit was used (in all book pages) at least 100 times. Find the least possible number of pages in the book. (Note: The book could have even as well as odd number of pages.)

A: 162 **B:** 163 **C:** 164 **D:** 165 **N:** None of these

Problem Sheet – 2003
6^{th} – 7^{th} Grades

1. Lottery tickets are numbered from 000000 till 999999 (different tickets have different numbers). Let's define a ticket as "lucky", if someone can obtain 0 using all its digits (in any order), parentheses, and addition / subtraction / multiplication operations. It's not necessary to use all operations or parentheses, but it's required to use each digit as many times as it appears in the ticket number. The usage of digits without any operation or parenthesis between them is prohibited. Find the number of "unlucky" tickets.

A: 0 **B:** 1000 **C:** 100000 **D:** 500000 **N:** None of these

2. Square 3×3 filled in with 9 integers (all these 9 numbers are different, and they are between 1 and 15). Let's multiply all 3 numbers within each row and each column, and obtain 6 products (some of them could be equal). Find the greatest possible number of equal products. For instance, if all 6 products could be equal to each other, the answer is 6, if only 5 products could be equal to each other, the answer is 5, and so on.

A: 2 **B:** 4 **C:** 5 **D:** 6 **N:** None of these

3. Find the number of all ways rectangle 2×15 could be cut into 15 dominoes (rectangles 2×1 or 1×2).

A: 789 **B:** 879 **C:** 897 **D:** 987 **N:** None of these

4. Let's define a one-way rook as a rook that can move / attack only in one direction (left, right, forward, or backward). In other words, a one-way rook is a rook with "face" that can move / attack only in the direction its "face" looks. Find the greatest possible number of one-way rooks that could be arranged on a standard 8×8 chessboard without attacking each other.

A: 24 **B:** 28 **C:** 30 **D:** 32 **N:** None of these

5. Find the number of all 6-step ways that satisfy the following conditions: the starting point is the bottom left vertex (corner) of a square, the ending point is the opposite vertex (corner) of the square, and each step is a move from one vertex (corner) to any of the two neighboring vertexes (corners) of the square (each vertex could appear multiple times in the same path).

A: 28 **B:** 30 **C:** 32 **D:** 36 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct addition example? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. NINE and SIX represent four-digit and three-digit numbers, so they cannot have 0 as their left digit.)

$$\begin{array}{rcccc} & N & I & N & E \\ + & & S & I & X \\ \hline 2 & 0 & 0 & 3 \end{array}$$

A: 0 **B:** 2 **C:** 4 **D:** 6 **N:** None of these

7. Find the missing number in the sequence: 1, 32, ?, 91, 121, 152, 182, ...

A: 56 **B:** 60 **C:** 61 **D:** 62 **N:** None of these

8. A "black box" contains 10 blue, 20 red, 30 green, and 40 white chips that are the same (except color). Find the least possible number of chips that should be randomly taken out of the "black box" to guarantee that at least 15 chips of the same color obtained.

A: 51 **B:** 57 **C:** 61 **D:** 63 **N:** None of these

9. There are 10 coins that look the same. Their weights (in ounces) are all different positive integers. If we arbitrarily put any 5 coins on the left plate and the remaining 5 coins on the right plate of correctly functioning scales (without any weights), the plate with the heaviest coin will weigh more than the other plate. Find the least possible weight (in ounces) of the heaviest coin.

A: 20 **B:** 26 **C:** 36 **D:** 100 **N:** None of these

10. All pages in a book are numbered in a standard way starting from the number 1. It appeared that at least 100 pages have the same product of their digits. Find the least possible number of pages in the book. (Note: The book could have even as well as odd number of pages.)

A: 520 **B:** 521 **C:** 530 **D:** 550 **N:** None of these

Problem Sheet – 2003
8th Grade

1. Lottery tickets are numbered from 000000 till 999999 (different tickets have different numbers). Let's define a ticket as "lucky", if someone can obtain 0 using all its digits (in any order), parentheses, and subtraction / multiplication operations. It's not necessary to use all operations or parentheses, but it's required to use each digit as many times as it appears in the ticket number. The usage of digits without any operation or parenthesis between them is prohibited. Find the number of "unlucky" tickets.

A: 0 **B:** 1000 **C:** 100000 **D:** 500000 **N:** None of these

2. Square 3×3 filled in with 9 integers (all these 9 numbers are different, and they are between 1 and 9). Let's multiply all 3 numbers within each row and each column, and obtain 6 products (some of them could be equal). Find the greatest possible number of equal products. For instance, if all 6 products could be equal to each other, the answer is 6, if only 5 products could be equal to each other, the answer is 5, and so on.

A: 2 **B:** 4 **C:** 5 **D:** 6 **N:** None of these

3. Find the number of all ways rectangle 3×15 could be cut into 15 triominoes (rectangles 3×1 or 1×3).

A: 188 **B:** 189 **C:** 198 **D:** 199 **N:** None of these

4. Let's consider isosceles triangles *ABC* (*AB* = *BC*), *BDC* (*BD* = *DC*), and *DEC* (*DE* = *EC*) that don't overlap each other, and points *A*, *C*, and *E* are on the same straight line. Find the least possible whole number *N* that satisfies the following condition: if at least *N* internal angles of the three isosceles triangles above are equal, then all these triangles are equilateral (it is not known which exactly *N* angles are equal).

A: 4 **B:** 5 **C:** 6 **D:** 7 **N:** None of these

5. Find the number of all 5-step ways that satisfy the following conditions: the starting point is a given vertex of a cube, the ending point is the opposite vertex of the cube, and each step is a move from one vertex to any of the three neighboring vertexes of the cube (each vertex could appear multiple times in the same path).

A: 48 **B:** 54 **C:** 60 **D:** 64 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct addition example? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. FOUR, SIX and TEN represent four-digit, three-digit and three-digit numbers, so they cannot have 0 as their left digit.)

```
  F O U R
+   S I X
+   T E N
-------
  2 0 0 3
```

A: 0 **B:** 1 **C:** 18 **D:** 36 **N:** None of these

7. Find the missing number in the sequence: 1, 3, ?, 4, 9, 3, 10, 2, 11, 1, ...

A: 0 **B:** 2 **C:** 4 **D:** 6 **N:** None of these

8. Sales tax in New York State is 8.25%. Cash registers automatically apply this tax to all taxable purchases (including Christmas tree toys) rounding tax amount to the nearest cent. For instance, when someone buys 2 toys ($1 each) the receipt will contain the following: item count (2), unit price ($1.00), subtotal (pre-tax) ($2.00), tax based on subtotal ($0.17), and total ($2.17). Find the greatest possible number of Christmas tree balls ($3 each) someone can buy in New York State spending at most $6500.00. No refunds, no returns.

A: 2000 **B:** 2001 **C:** 2002 **D:** 2003 **N:** None of these

9. There are 10 coins that look the same. Their weights (in ounces) are all different positive integers. If we arbitrarily put any 2 coins on the left plate and any 2 (from the remaining 8) coins on the right plate of correctly functioning scales (without any weights), the plate with the heaviest (out of 4 coins taken) coin will weigh more than the other plate. Find the least possible weight (in ounces) of the heaviest (out of all 10 coins) coin.

A: 80 **B:** 81 **C:** 88 **D:** 89 **N:** None of these

10. All pages in a book are numbered in a standard way starting from the number 1. It appeared that at least 100 pages have the same product of their digits. Find the least possible number of pages in the book. (Note: The book could have even as well as odd number of pages.)

A: 520 **B:** 521 **C:** 530 **D:** 550 **N:** None of these

Problem Sheet – 2003

9^{th} – 10^{th} Grades

1. Lottery tickets are numbered from 000000 till 999999 (different tickets have different numbers). Let's define a ticket as "lucky", if someone can obtain 0 using all its digits (in any order), parentheses, and subtraction / division operations. It's not necessary to use all operations or parentheses, but it's required to use each digit as many times as it appears in the ticket number. The usage of digits without any operation or parenthesis between them is prohibited as well as the usage of "invisible" multiplication operation. Find the number of "unlucky" tickets.

A: 0 **B:** 1000 **C:** 100000 **D:** 500000 **N:** None of these

2. Square 3×3 filled in with 9 integers (all these 9 numbers are different, and they are between 1 and *N*). Let's multiply all 3 numbers within each row and each column, and obtain 6 products. It appeared that all these 6 products are equal to each other. Find the least possible value of *N*.

A: 20 **B:** 21 **C:** 22 **D:** 24 **N:** None of these

3. Find the number of all ways rectangle 3×10 could be cut into 15 dominoes (rectangles 2×1 or 1×2).

A: 500 **B:** 517 **C:** 527 **D:** 571 **N:** None of these

4. Let's consider isosceles triangles *ABC* (*AB* = *BC*), *BDC* (*BD* = *DC*), and *DEC* (*DE* = *EC*) that don't overlap each other, and points *A*, *C*, and *E* are on the same straight line. Find the least possible whole number *N* that satisfies the following condition: if at least *N* internal angles of the three isosceles triangles above are equal, then all these triangles are equilateral (it is not known which exactly *N* angles are equal).

A: 4 **B:** 5 **C:** 6 **D:** 7 **N:** None of these

5. Find the number of all 7-step ways that satisfy the following conditions: the starting point is the bottom left vertex of a square, the ending point is the bottom right vertex of the square, and each step is a move from one vertex to any of the two neighboring vertexes of the square (each vertex could appear multiple times in the same path).

A: 48 **B:** 49 **C:** 60 **D:** 64 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct addition example? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. FOUR, SIX and TEN represent four-digit, three-digit and three-digit numbers, so they cannot have 0 as their left digit.)

```
  F O U R
+   S I X
+   T E N
-------
  2 0 0 3
```

A: 0 **B:** 1 **C:** 18 **D:** 36 **N:** None of these

7. Find the missing number in the sequence: 1, 2, 12, ?, 75600, ...

A: 300 **B:** 360 **C:** 365 **D:** 480 **N:** None of these

8. Sales tax in New York State is 8.25%. Cash registers automatically apply this tax to all taxable purchases (including Christmas tree toys) rounding tax amount to the nearest cent. For instance, when someone buys 2 toys ($1 each) the receipt will contain the following: item count (2), unit price ($1.00), subtotal (pre-tax) ($2.00), tax based on subtotal ($0.17), and total ($2.17). Find the greatest possible number of Christmas tree balls ($3 each) someone can buy in New York State spending at most $6500.00. No refunds, no returns.

A: 2000 **B:** 2001 **C:** 2002 **D:** 2003 **N:** None of these

9. The two non-parallel sides of a given parallelogram (that is not a square) are in the same ratio as its diagonals. Find $c \div a$, where *c* and *a* are the longest diagonal and side of this parallelogram respectively.

A: $\sqrt{2}$ **B:** 1.5 **C:** $(\sqrt{5} + 1)/2$ **D:** 2 **N:** None of these

10. All pages in a book are numbered in a standard way starting from the number 1. It appeared that at least 100 pages have the same sum of their digits. Find the least possible number of pages in the book. (Note: The book could have even as well as odd number of pages.)

A: 1281 **B:** 1282 **C:** 1290 **D:** 1308 **N:** None of these

Problem Sheet – 2003
11^{th} – 12^{th} Grades

1. Lottery tickets are numbered from 111111 till 999999 (different tickets have different numbers, ticket numbers do not contain 0). Let's define a ticket as "lucky", if someone can obtain a whole number using all its digits (in any order), parentheses, and division operations. It's not necessary to use parentheses, but it's required to use each digit as many times as it appears in the ticket number. The usage of digits without any operation or parenthesis between them is prohibited as well as the usage of "invisible" multiplication operation. Find the number of "unlucky" tickets.

A: 0 **B:** 1000 **C:** 100000 **D:** 500000 **N:** None of these

2. Find the greatest possible number of k that satisfies the following condition: for any triangle with sides a, b, and c, it is possible to make up a triangle with sides $ka^2 + bc$, $kb^2 + ca$, and $kc^2 + ab$.

A: 1 **B:** 1.5 **C:** 1⅔ **D:** 1.75 **N:** None of these

3. Find the last (the most right) digit of F_{2003}, where $\{F_n\}$ is the Fibonacci sequence ($F_1 = 1$, $F_2 = 1$, $F_{n+2} = F_n + F_{n+1}$ for any nonnegative integer n).

A: 1 **B:** 3 **C:** 5 **D:** 7 **N:** None of these

4. Each side of a given triangle with area 2003^2 is divided into 2003 equal segments. For any two distinct points out of 6009 endpoints of these segments let's draw a straight line containing these two points. Each of these lines cuts the given triangle into two figures. Let's define a *balance* of the line as an absolute value of difference of areas of those figures. Find the least possible value of the *balance*.

A: 0.5 **B:** 1 **C:** 3 **D:** 2003 **N:** None of these

5. Find the number of all 6-step ways that satisfy the following conditions: the starting point is a given bottom vertex of a cube, the ending point is the opposite bottom vertex of the cube, and each step is a move from one vertex to any of the three neighboring vertexes of the cube (each vertex could appear multiple times in the same path).

A: 177 **B:** 180 **C:** 181 **D:** 182 **N:** None of these

6. In how many different ways can you replace letters with digits to obtain correct addition example? (Note: Each letter represents a digit, different letters represent different digits, and the same letters represent the same digits. FOUR, SIX and TEN represent four-digit, three-digit and three-digit numbers, so they cannot have 0 as their left digit.)

```
  F O U R
+   S I X
+   T E N
-------
  2 0 0 3
```

A: 0 **B:** 1 **C:** 18 **D:** 36 **N:** None of these

7. Find the missing number in the sequence: ?, 23, 29, 41, 43, 47, 53, 59, ...

A: 11 **B:** 13 **C:** 15 **D:** 17 **N:** None of these

8. Sales tax in New York State is 8.25%. Cash registers automatically apply this tax to all taxable purchases (including Christmas tree toys) rounding tax amount to the nearest cent. For instance, when someone buys 2 toys ($1 each) the receipt will contain the following: item count (2), unit price ($1.00), subtotal (pre-tax) ($2.00), tax based on subtotal ($0.17), and total ($2.17). Find the greatest possible number of Christmas tree balls ($3 each) someone can buy in New York State spending at most $6500.00. No refunds, no returns.

A: 2000 **B:** 2001 **C:** 2002 **D:** 2003 **N:** None of these

9. The two non-parallel sides of a given parallelogram (that is not a square) are in the same ratio as its diagonals. Find $c \div a$, where c and a are the longest diagonal and side of this parallelogram respectively.

A: $\sqrt{2}$ **B:** 1.5 **C:** $(\sqrt{5} + 1)/2$ **D:** 2 **N:** None of these

10. All pages in a book are numbered in a standard way starting from the number 1. It appeared that at least 100 pages have the same sum of their digits. Find the least possible number of pages in the book. (Note: The book could have even as well as odd number of pages.)

A: 1281 **B:** 1282 **C:** 1290 **D:** 1308 **N:** None of these

Answer Sheet – 2003

4^{th} – 5^{th} Grades

1. ● B C D N
2. A B C ● N
3. A B ● D N
4. A ● C D N
5. A B ● D N
6. ● B C D N
7. A B ● D ●
8. A B C D ●
9. A B ● D N
10. ● B C D N

Total Score: 50

Answer Sheet – 2003

6^{th} – 7^{th} Grades

1. ● B C D N
2. A B C ● N
3. A B C ● N
4. A ● C D N
5. A B ● D N
6. ● B C D N
7. A ● C ● N
8. A B C D ●
9. A ● C D N
10. A B C ● N

Total Score: 50

Answer Sheet – 2003

8th Grade

1. ● B C D N
2. A B C D ●
3. A ● C D N
4. A ● C D N
5. A B ● D N
6. ● B C D N
7. ● B C D N
8. A B ● D N
9. A B C ● N
10. A B C ● N

Total Score: 50

Answer Sheet – 2003

9^{th} – 10^{th} Grades

1. ● B C D N
2. A B C D ●
3. A B C ● N
4. A ● C D N
5. A B C ● N
6. ● B C D N
7. A ● C D N
8. A B ● D N
9. ● B C D N
10. A B ● D N

Total Score: 50

Answer Sheet – 2003

11^{th} – 12^{th} Grades

1. ● B C D N
2. A ● C D N
3. A B C ● N
4. A ● C D N
5. A B C ● N
6. ● B C D N
7. A B C D ●
8. A B ● D N
9. ● B C D N
10. A B ● D N

Total Score: 50