Problem Sheet – 2002

9^{th} – 10^{th} Grades

1. Compute $$\frac{1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \ldots + 2000\cdot 2001 + 2001\cdot 2002}{1^2 + 3^2 + 5^2 + \ldots + 1999^2 + 2001^2}$$

A: 2 **B:** 3 **C:** 1001 **D:** 2002 **N:** None of these

2. Segment *AM* cuts a given triangle *ABC* into two triangles (*ABM* and *ACM*) that are similar but not equal, *BM* = 16 cm, *CM* = 9 cm. Find the length of the segment *AB*.

A: 16 cm **B:** 20 cm **C:** 24 cm **D:** 25 cm **N:** None of these

3. Find the greatest possible number of consecutive ones that could appear at the beginning of the decimal representation of a perfect square number.

A: 2 **B:** 10 **C:** 100 **D:** 2002 **N:** None of these

4. Segment *DE* cuts a given triangle *ABC* into triangle *DBE* and quadrilateral *ADEC*, *BD*:*DA* = 2:1, *BE*:*EC* = 3:2. Find the ratio of the area of triangle *DBE* to the area of quadrilateral *ADEC*.

A: 1:2 **B:** 3:5 **C:** 5:8 **D:** 2:3 **N:** None of these

5. A new computer game has 10 levels (level 0, level 1, ..., level 9), and its hero has three lives. The game starts at the level 0. Each time the current level is successfully finished the game continues at the next level. First or second time the hero dies the game continues at the previous level (unless he dies at the level 0). The game is over if level 9 is successfully finished, or hero dies third time, or hero dies at the level 0. After the game is over computer prints the game history, i.e. sequence of levels the hero played, and the last level he successfully finished (if any). For instance, sequence 0, 1, 0, 1, 2, 1, 1 means that hero successfully finished level 0, died at level 1, successfully finished levels 0 and 1 and died at levels 2 and 1. Find the greatest possible number of different game histories.

A:287 **B:** 288 **C:** 309 **D:** 327 **N:** None of these

6. Let's consider sets of red, blue and white chips (each set can contain any number of chips of any of these three colors). Let's define a set as a beautiful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors. Let's define a set as a wonderful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors, and any two chips with exactly two neighboring chips between them have different colors. Find such a number M that any beautiful set of M chips is also a wonderful one.

A: 8 **B:** 10 **C:** 12 **D:** 100 **N:** None of these

```
  S E V E N
+     O N E
-----------
  E I G H T
```

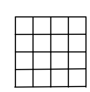
7. Find the missing number in the sequence: 1, 2, 2, 3, ?, 4, 2, 4, 3, 4.

A: 1 **B:** 2 **C:** 3 **D:** 4 **N:** None of these

8. In how many different ways can you replace letters with digits to obtain correct addition example? (Note: Each letter represents a digit, different letters represent different digits, the same letters represent the same digits. SEVEN, ONE and EIGHT represent five-digit, three-digit and five-digit numbers, so they cannot have 0 as their left digit.)

A: 0 **B:** 1 **C:** 2 **D:** 3 **N:** None of these

9. Square 4×4 cut into 16 unit cells. Some of their vertexes are marked in such a way that any straight line contains no more than two marked vertexes. Find the greatest possible number of marked vertexes.



A: 8 **B:** 9 **C:** 10 **D:** 11 **N:** None of these

10. There are 50 students in a class. Each of them is taking Biology, Chemistry or Physics (some students could take more than one class). 42 students are taking Physics or Chemistry, 41 – Physics or Biology, 40 – Chemistry or Biology, 28 – Physics, and 26 – Chemistry. Find the greatest possible number of students taking Biology.

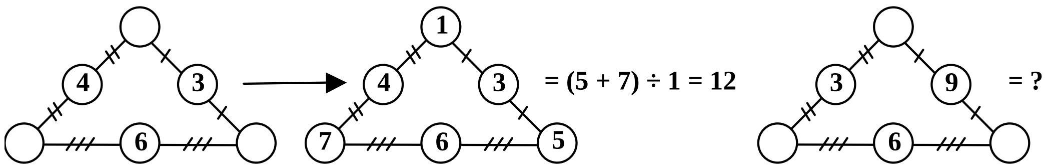
A: 25 **B:** 30 **C:** 31 **D:** 32 **N:** None of these

Problem Sheet – 2002
4^{th} – 5^{th} Grades

1. A triangle cut into quadrilaterals. Find the least possible number of these quadrilaterals.

A: 1 **B:** 2 **C:** 3 **D:** 4 **N:** None of these

2. Using the triangular pattern below, compute the value represented by the right triangle.



A: 2 **B:** 3 **C:** 4 **D:** 5 **N:** None of these

3. 50 students are sitting around the table, and they are 'marked' with tags 1, 2, 3, ..., 50 clockwise. They are playing the following game: one of them (not necessary the student No. 1) says 'One', then his neighbor clockwise says 'Two', then his neighbor clockwise says 'Three', and so on. Any student who says the number divisible by 5 leaves the game (together with his chair). The game is over when there is only one student left, and this student becomes a winner. Finally, the student No. 50 won the game. Find the tag of the student who started the game.

A: 16 **B:** 19 **C:** 23 **D:** 32 **N:** None of these

4. Imagine that some country has only two kinds of coins: 7 cents and 11 cents, and does not have any bills. It has also a special law that prohibits to sell anything if its price could be paid without a change (any price should be a whole number of cents). Find the greatest possible (according to the law) price in this country.

A: 52 **B:** 58 **C:** 59 **D:** 60 **N:** None of these

5. A new computer game has 10 levels (level 0, level 1, ..., level 9), and its hero has two lives. The game starts at the level 0. Each time the current level is successfully finished the game continues at the next level. First time the hero dies the game continues at the previous level (unless he dies at the level 0). The game is over if level 9 is successfully finished, or hero dies second time, or hero dies at the level 0. After the game is over computer prints the game history, i.e. sequence of levels the hero played, and the last level he successfully finished (if any). For instance, sequence 0, 1, 0, 1, 2, 1 means that hero successfully finished level 0, died at level 1, successfully finished levels 0 and 1 and died at level 2. Find the greatest possible number of different game histories.

A: 63 **B:** 64 **C:** 65 **D:** 70 **N:** None of these

6. Let's consider sets of red, blue and white chips (each set can contain any number of chips of any of these three colors). Let's define a set as a beautiful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors. Let's define a set as a wonderful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors, and any two chips with exactly two neighboring chips between them have different colors. Find such a number M that any beautiful set of M chips is also a wonderful one.

A: 11 **B:** 13 **C:** 15 **D:** 17 **N:** None of these

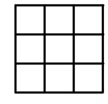
7. Find the missing number in the sequence: 1, 9, 71, ?, 33, 14, 94, 75.

A: 12 **B:** 49 **C:** 52 **D:** 99 **N:** None of these

8. Compute 1 + 4 + 7 + ... + 1996 + 1999 + 2002 – 2 – 5 – 8 – ... – 1994 – 1997 – 2000.

A: 667 **B:** 1335 **C:** 2001 **D:** 2002 **N:** None of these

9. Square 3×3 cut into 9 unit cells. Some of their vertexes are marked in such a way that any straight line contains no more than two marked vertexes. Find the greatest possible number of marked vertexes.



A: 5 **B:** 7 **C:** 9 **D:** 10 **N:** None of these

10. A dealer is selling 6 brand new cars: red and white Hondas, red and white Nissans and red and white Toyotas. White Honda costs \$2000 more than red Nissan. Red Honda costs \$500 more than white Toyota. Find how much white Nissan costs more than red Toyota if each white car costs \$1000 more than the corresponding red car.

A: \$500 **B:** \$1000 **C:** \$1500 **D:** \$2000 **N:** None of these

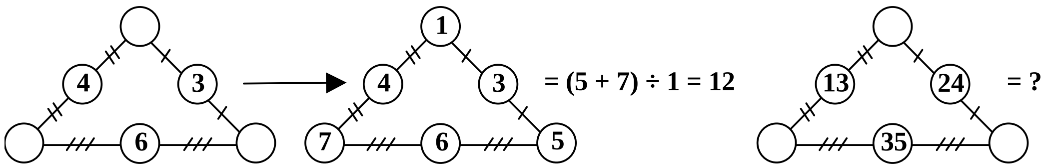
Problem Sheet – 2002

6^{th} – 7^{th} Grades

1. A triangle cut into pentagons. Find the least possible number of these pentagons.

A: 4 **B:** 5 **C:** 7 **D:** 10 **N:** None of these

2. Using the triangular pattern below, compute the value represented by the right triangle.



A: 35 **B:** 45 **C:** 48 **D:** 52 **N:** None of these

3. 100 students are sitting around the table, and they are 'marked' with tags 1, 2, 3, ..., 100 clockwise. They are playing the following game: one of them (not necessary the student No. 1) says 'One', then his neighbor clockwise says 'Two', then his neighbor clockwise says 'Three', and so on. Any student who says the number divisible by 5 leaves the game (together with his chair). The game is over when there is only one student left, and this student becomes a winner. Finally, the student No. 100 won the game. Find the tag of the student who started the game.

A: 60 **B:** 61 **C:** 62 **D:** 72 **N:** None of these

4. Imagine that some country has only two kinds of coins: 9 cents and 13 cents, and does not have any bills. It has also a special law that prohibits to sell anything if its price could be paid without a change (any price should be a whole number of cents). Find the greatest possible (according to the law) price in this country.

A: 86 **B:** 94 **C:** 95 **D:** 96 **N:** None of these

5. A new computer game has 20 levels (level 0, level 1, ..., level 19), and its hero has two lives. The game starts at the level 0. Each time the current level is successfully finished the game continues at the next level. First time the hero dies the game continues at the previous level (unless he dies at the level 0). The game is over if level 19 is successfully finished, or hero dies second time, or hero dies at the level 0. After the game is over computer prints the game history, i.e. sequence of levels the hero played, and the last level he successfully finished (if any). For instance, sequence 0, 1, 0, 1, 2, 1 means that hero successfully finished level 0, died at level 1, successfully finished levels 0 and 1 and died at level 2. Find the greatest possible number of different game histories.

A: 221 **B:** 228 **C:** 229 **D:** 230 **N:** None of these

6. Let's consider sets of red, blue and white chips (each set can contain any number of chips of any of these three colors). Let's define a set as a beautiful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors. Let's define a set as a wonderful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors, and any two chips with exactly two neighboring chips between them have different colors. Find such a number M that any beautiful set of M chips is also a wonderful one.

A: 11 **B:** 13 **C:** 15 **D:** 17 **N:** None of these

7. Find the missing number in the sequence: 1, 9, 18, 927, ?, 94095.

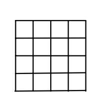
A: 1656 **B:** 2002 **C:** 2007 **D:** 8361 **N:** None of these

```
   F I V E
+    T W O
----------
 S E V E N
```

8. In how many different ways can you replace letters with digits to obtain correct addition example? (Note: Each letter represents a digit, different letters represent different digits, the same letters represent the same digits. FIVE, TWO and SEVEN represent four-digit, three-digit and five-digit numbers, so they cannot have 0 as their left digit.)

A: 0 **B:** 1 **C:** 2 **D:** 3 **N:** None of these

9. Square 4×4 cut into 16 unit cells. Some of their vertexes are marked in such a way that any vertical or horizontal straight line contains no more than three marked vertexes. Find the greatest possible number of marked vertexes.



A: 12 **B:** 14 **C:** 16 **D:** 18 **N:** None of these

10. A store is selling beef in three different packs using the following price policy: 1-lb pack costs \$3, 5-lb pack costs \$11, and 13-lb pack costs \$28. What is the least amount needed to buy exactly 100 pounds of beef in this store?

A: \$215 **B:** \$217 **C:** \$219 **D:** \$221 **N:** None of these

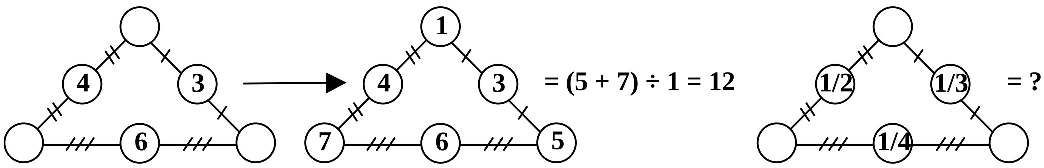
Problem Sheet – 2002

8^{th} Grade

1. A decagon cut into 2002-gons. Find the least possible number of these 2002-gons.

A: 10 **B:** 20 **C:** 2001 **D:** 2002 **N:** None of these

2. Using the triangular pattern below, compute the value represented by the right triangle.



A: 3/4 **B:** 4/5 **C:** 5/6 **D:** 6/7 **N:** None of these

3. There is a set of 6 segments. One of these segments should be removed in order to make a pentagon from the remaining segments (they become pentagon's sides). It is possible to do if segment lengths (in cm) are:

A: 1,1,2,4,8,16 **B:** 1,2,3,4,9,18 **C:** 2,2,2,3,9,16 **D:** 1,2,3,3,9,17 **N:** None of these

4. Imagine that some country has only two kinds of coins: 9 cents and 14 cents, and does not have any bills. It has also a special law that prohibits to sell anything if its price could be paid without a change (any price should be a whole number of cents). Find the greatest possible (according to the law) price in this country.

A: 89 **B:** 94 **C:** 103 **D:** 104 **N:** None of these

5. A new computer game has 5 levels (level 0, level 1, ..., level 4), and its hero has three lives. The game starts at the level 0. Each time the current level is successfully finished the game continues at the next level. First or second time the hero dies the game continues at the previous level (unless he dies at the level 0). The game is over if level 4 is successfully finished, or hero dies third time, or hero dies at the level 0. After the game is over computer prints the game history, i.e. sequence of levels the hero played, and the last level he successfully finished (if any). For instance, sequence 0, 1, 0, 1, 2, 1, 1 means that hero successfully finished level 0, died at level 1, successfully finished levels 0 and 1 and died at levels 2 and 1. Find the greatest possible number of different game histories.

A:42 **B:** 52 **C:** 62 **D:** 72 **N:** None of these

6. Let's consider sets of red, blue and white chips (each set can contain any number of chips of any of these three colors). Let's define a set as a beautiful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors. Let's define a set as a wonderful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors, and any two chips with exactly two neighboring chips between them have different colors. Find such a number M that any beautiful set of M chips is also a wonderful one.

A: 6 **B:** 8 **C:** 10 **D:** 12 **N:** None of these

7. Find the missing number in the sequence: 1, 2, 3, 2, 5, 3, ?, 2, 3, 5.

A: 2 **B:** 3 **C:** 7 **D:** 9 **N:** None of these

```
   F I V E
+    T W O
----------
 S E V E N
```

8. In how many different ways can you replace letters with digits to obtain correct addition example? (Note: Each letter represents a digit, different letters represent different digits, the same letters represent the same digits. FIVE, TWO and SEVEN represent four-digit, three-digit and five-digit numbers, so they cannot have 0 as their left digit.)

A: 0 **B:** 1 **C:** 2 **D:** 3 **N:** None of these

9. Find the greatest number of different results that could be obtained by putting any number of parentheses in the expression 2 ÷ 3 ÷ 5 ÷7 ÷11 ÷13 ÷17.

A: 16 **B:** 24 **C:** 30 **D:** 32 **N:** None of these

10. There are 50 students in a class. Each of them is taking Biology, Chemistry or Physics (some students could take more than one class). 42 students are taking Physics or Chemistry, 41 – Physics or Biology, 40 – Chemistry or Biology, 28 – Physics, and 26 – Chemistry. Find the least possible number of students taking Biology.

A: 18 **B:** 19 **C:** 20 **D:** 21 **N:** None of these

Problem Sheet – 2002
11^{th} – 12^{th} Grades

1. Compute $$\frac{1\cdot 2 + 2\cdot 3 + 3\cdot 4 + ... + 2000\cdot 2001 + 2001\cdot 2002}{1^2 + 3^2 + 5^2 + ... + 1999^2 + 2001^2}$$

A: 2 **B:** 3 **C:** 1001 **D:** 2002 **N:** None of these

2. Segment *AM* cuts a given triangle *ABC* into two triangles (*ABM* and *ACM*) that are similar but not equal, *BM* = 16 cm, *CM* = 9 cm. Find the length of the segment *AB*.

A: 16 cm **B:** 20 cm **C:** 24 cm **D:** 25 cm **N:** None of these

3. Find the greatest possible number of consecutive ones that could appear at the beginning of the decimal representation of a perfect cube number.

A: 2 **B:** 10 **C:** 100 **D:** 2002 **N:** None of these

4. How many 9-digit numbers satisfy the following condition: if the number appends to itself then the resulting 18-digit number is divisible by 2002.

A: $5\cdot 10^7$ **B:** $45\cdot 10^7$ **C:** $6\cdot 10^8$ **D:** $9\cdot 10^8$ **N:** None of these

5. A new computer game has 10 levels (level 0, level 1, ..., level 9), and its hero has three lives. The game starts at the level 0. Each time the current level is successfully finished the game continues at the next level. First or second time the hero dies the game continues at the previous level (unless he dies at the level 0). The game is over if level 9 is successfully finished, or hero dies third time, or hero dies at the level 0. After the game is over computer prints the game history, i.e. sequence of levels the hero played, and the last level he successfully finished (if any). For instance, sequence 0, 1, 0, 1, 2, 1, 1 means that hero successfully finished level 0, died at level 1, successfully finished levels 0 and 1 and died at levels 2 and 1. Find the greatest possible number of different game histories.

A:287 **B:** 288 **C:** 309 **D:** 327 **N:** None of these

6. Let's consider sets of red, blue and white chips (each set can contain any number of chips of any of these three colors). Let's define a set as a beautiful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors. Let's define a set as a wonderful set if all its chips could be arranged around a circle in such a way that any two neighboring chips have different colors, and any two chips with exactly two neighboring chips between them have different colors. Find such a number M that any beautiful set of M chips is also a wonderful one.

A: 8 **B:** 10 **C:** 12 **D:** 2002 **N:** None of these

7. Find the missing number in the sequence: 1, 1, 2, 2, ?, 2, 6, 4, 6, 4.

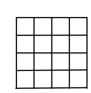
A: 1 **B:** 2 **C:** 4 **D:** 6 **N:** None of these

$$\begin{array}{cccc} & F & O & U & R \\ + & F & I & V & E \\ \hline & N & I & N & E \end{array}$$

8. In how many different ways can you replace letters with digits to obtain correct addition example? (Note: Each letter represents a digit, different letters represent different digits, the same letters represent the same digits. FOUR, FIVE and NINE represent four-digit numbers, so they cannot have 0 as their left digit.)

A: 64 **B:** 72 **C:** 76 **D:** 96 **N:** None of these

9. Square 4×4 cut into 16 unit cells. Some of their vertexes are marked in such a way that any straight line contains no more than two marked vertexes. Find the greatest possible number of marked vertexes.



A: 8 **B:** 9 **C:** 10 **D:** 11 **N:** None of these

10. Imagine that some clock has identical hour-hand and minute-hand, and it does not have a second-hand. Find how many times per twenty-four hours it is impossible to distinguish between its hour-hand and minute-hand. (Note: The clock always shows correct time. For this problem consider twenty-four hours starting at 6:00 am.)

A: 22 **B:** 132 **C:** 144 **D:** 286 **N:** None of these

Answer Sheet – 2002

4th – 5th Grades

1.	Ⓐ	●	Ⓒ	Ⓓ	Ⓝ
2.	●	Ⓑ	Ⓒ	Ⓓ	Ⓝ
3.	Ⓐ	Ⓑ	Ⓒ	●	Ⓝ
4.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ
5.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	●
7.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ
8.	Ⓐ	●	Ⓒ	Ⓓ	Ⓝ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	●
10.	Ⓐ	Ⓑ	●	Ⓓ	Ⓝ

Total Score: 50

Answer Sheet – 2002

6th – 7th Grades

1. Ⓐ Ⓑ Ⓒ Ⓓ ●
2. ● Ⓑ Ⓒ Ⓓ Ⓝ
3. Ⓐ Ⓑ Ⓒ ● Ⓝ
4. Ⓐ Ⓑ ● Ⓓ Ⓝ
5. Ⓐ Ⓑ Ⓒ ● Ⓝ
6. Ⓐ Ⓑ Ⓒ Ⓓ ●
7. ● Ⓑ Ⓒ Ⓓ Ⓝ
8. ● Ⓑ Ⓒ Ⓓ Ⓝ
9. Ⓐ Ⓑ Ⓒ Ⓓ ●
10. Ⓐ ● Ⓒ Ⓓ Ⓝ

Total Score: 50

Answer Sheet – 2002

8th Grade

1.	A	B	C	D	●
2.	A	B	C	●	N
3.	A	●	C	D	N
4.	A	B	●	D	N
5.	A	B	●	D	N
6.	A	B	C	D	●
7.	A	B	●	D	N
8.	●	B	C	D	N
9.	A	B	C	●	N
10.	A	●	C	D	N

Total Score: 50

Answer Sheet – 2002

9^{th} – 10^{th} Grades

1.	●	B	C	D	N
2.	A	●	C	D	N
3.	A	B	C	D	●
4.	A	B	C	●	N
5.	A	B	C	●	N
6.	A	B	C	●	N
7.	A	●	C	D	N
8.	●	B	C	D	N
9.	A	B	●	D	N
10.	A	B	●	D	N

Total Score: 50

Answer Sheet – 2002

11^{th} – 12^{th} Grades

1.	●	B	C	D	N
2.	A	●	C	D	N
3.	A	B	C	D	●
4.	A	●	C	D	N
5.	A	B	C	●	N
6.	A	B	C	●	N
7.	A	B	●	D	N
8.	A	●	C	D	N
9.	A	B	●	D	N
10.	A	B	C	●	N

Total Score: 50