

# Problem Sheet – 2000

## 4<sup>th</sup> – 5<sup>th</sup> Grades

1. Find the greatest possible number of September weekend days in a year.

A: 8            B: 9            C: 10            D: 11            N: None of these

2. Square  $3 \times 3$  filled in with 9 numbers. The sum of the numbers in the first row is equal to 80, in the second – 90, in the third – 100. The sum of the numbers in the first column is equal to 110, in the second – 120. Find the sum of the numbers in the third column.

A: 10            B: 40            C: 70            D: 130            N: None of these

3. What is the next number in the sequence: 1, 2, 4, 8, 16, 23, 28, 38, ...?

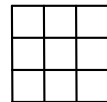
A: 43            B: 47            C: 48            D: 49            N: None of these

4. There are two types of residents on the planet called Z. Any resident of the first type has 1 head, 5 arms and 3 legs, and any resident of the second type has 2 arms, 4 legs and 6 heads. All employees of The Bureau of Interplanetary Contacts of the planet Z have a total of 220 arms and heads. How many legs do they have altogether?

A: 70            B: 110            C: 119            D: 120            N: None of these

5. How many rectangles are in this figure?

A: 18            B: 22            C: 32            D: 36            N: None of these



6. Find the last digit of  $3 \cdot 3 \cdot 3 \cdot \dots \cdot 3 \cdot 3 \cdot 3$  (2000 factors).

A: 3            B: 5            C: 7            D: 9            N: None of these

7. How many different ways are there to pay \$19 without change using bills \$1, \$2, \$5 and \$10? It is not necessary to use all kinds of these bills.

A: 16            B: 24            C: 34            D: 42            N: None of these

8. There are several mines in the cells of  $4 \times 4$  field (each cell contains either zero mines or one mine). Four cells are 'opened' (they have no mines), and they have numbers representing the amount of mines in neighboring cells (each cell has maximum 8 neighboring cells). In how many different ways can mines be arranged in this field?

3			
		5	
	5		
			3

A: 3            B: 7            C: 8            D: 9            N: None of these

9. How many whole numbers between 1 and 7500 that are divisible by 3 but not divisible by 5?

A: 1000            B: 1500            C: 2000            D: 2500            N: None of these

10. All whole numbers are written in increasing order. All their digits are separated from each other with comas. What number is in the 2000<sup>th</sup> place in the made sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 0, 1, 1, 1, 2, 1, 3, ...?

A: 0            B: 1            C: 3            D: 7            N: None of these

# Problem Sheet – 2000

## 6<sup>th</sup> – 7<sup>th</sup> Grades

1. Let  $A$  be the least integer with the sum of all its digits is equal to 2000. Find the left digit of  $A$ .

- A: 1      B: 3      C: 4      D: 6      N: None of these

2. Square  $3 \times 3$  filled in with 9 numbers. The product of the numbers in the first row is equal to 60, in the second – 80, in the third – 120. The product of the numbers in the first column is equal to 90, in the second – 100. Find the product of the numbers in the third column.

- A: 32      B: 40      C: 48      D: 64      N: None of these

3. Three equilateral triangles that are not necessary equal were cut from the given equilateral triangle. These cut triangles have no common points, but each of them has common vertex with the given triangle. Find the perimeter of the remaining hexagon if the perimeter of the given triangle is 100 cm and the sum of three cut triangles' perimeters is equal to 90 cm.

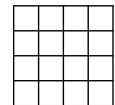
- A: 55 cm      B: 70 cm      C: 74 cm      D: 75 cm      N: None of these

4. There are two types of residents on the planet called Z. Any resident of the first type has 1 head, 5 arms and 3 legs, and any resident of the second type has 2 arms, 4 legs and 6 heads. All employees of The Bureau of Interplanetary Contacts of the planet Z have a total of 220 arms and heads. How many legs do they have altogether?

- A: 70      B: 110      C: 119      D: 120      N: None of these

5. How many rectangles are in this figure?

- A: 48      B: 70      C: 100      D: 120      N: None of these



6. Find the last digit of  $7^{2000}$ .

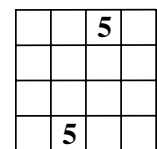
- A: 3      B: 5      C: 7      D: 9      N: None of these

7. How many different ways are there to pay 50 cents without change using coins 1 cent, 5 cents, 10 cents and 25 cents? It is not necessary to use all kinds of these coins.

- A: 16      B: 25      C: 49      D: 64      N: None of these

8. There are several mines in the cells of  $4 \times 4$  field (each cell contains either zero mines or one mine). Two cells are 'opened' (they have no mines), and they have numbers representing the amount of mines in neighboring cells (each cell has maximum 8 neighboring cells). In how many different ways can mines be arranged in this field?

- A: 4      B: 8      C: 12      D: 16      N: None of these



9. How many whole numbers less than 4500 are divisible neither by 3 nor by 5?

- A: 2000      B: 2100      C: 2400      D: 2600      N: None of these

10. All whole numbers are written in increasing order. All their digits are separated from each other with comas. What number is in the 2000<sup>th</sup> place in the made sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 0, 1, 1, 1, 2, 1, 3, ...?

- A: 0      B: 1      C: 3      D: 7      N: None of these

# Problem Sheet – 2000

## 8<sup>th</sup> Grade

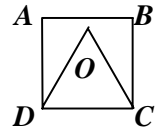
1. Imagine you cut cubed kilometer of wood into cubed meters and put them in one line to build a road which width is 1 meter. What is the length of this road?

A:  $10^3$  km    B:  $10^6$  km    C:  $10^8$  km    D:  $10^9$  km    N: None of these

2. Square  $3 \times 3$  filled in with 9 numbers. The product of the numbers in the first row is equal to 60, in the second – 80, in the third – 120. The product of the numbers in the first column is equal to 90, in the second – 100. Find the product of the numbers in the third column.

A: 32    B: 40    C: 48    D: 64    N: None of these

3. An equilateral triangle  $COD$  is placed inside a square  $ABCD$  as shown in the picture. Find the measure of angle  $OAB$ .

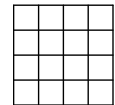


A:  $10^\circ$     B:  $15^\circ$     C:  $20^\circ$     D:  $21^\circ$     N: None of these

4. How many 5-digit whole numbers satisfy the following condition: the product of five digits of the number is equal to 2000?

A: 10    B: 20    C: 24    D: 30    N: None of these

5. How many rectangles are in this figure?



A: 48    B: 70    C: 100    D: 120    N: None of these

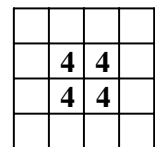
6. Find the next to last digit of  $6^{2000}$ .

A: 1    B: 3    C: 5    D: 9    N: None of these

7. You have 6 bags with coins that look the same. Each bag has infinite number of coins. Coins from one of bags weigh 1 gram each, coins from another bag weigh 2 grams each, ..., coins from the last bag weigh 6 grams each. The tag number (1, 2, 3, 4, 5, 6) on the top of each bag should correspond to the weight of any coin from this bag. You have scales without any weights and without a scale. What is the least number of attempts to weigh you should make to be sure that all tag numbers are correct?

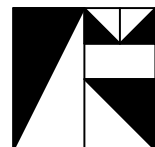
A: 1    B: 2    C: 3    D: 4    N: None of these

8. There are several mines in the cells of  $4 \times 4$  field (each cell contains either zero mines or one mine). Four cells are 'opened' (they have no mines), and they have numbers representing the amount of mines in neighboring cells (each cell has maximum 8 neighboring cells). In how many different ways can mines be arranged in this field?



A: 10    B: 15    C: 17    D: 20    N: None of these

9. What fractional part of the figure is shaded?



A:  $7/32$     B:  $7/16$     C:  $4/9$     D:  $1/2$     N: None of these

10. Compute the following product:  $\left(1 - \frac{1}{4}\right) \cdot \left(1 - \frac{1}{9}\right) \cdot \left(1 - \frac{1}{16}\right) \cdots \left(1 - \frac{1}{9801}\right) \cdot \left(1 - \frac{1}{10000}\right)$ .

A: 0.404    B: 0.5    C: 0.505    D: 0.65    N: None of these

# Problem Sheet – 2000

## 9<sup>th</sup> – 10<sup>th</sup> Grades

1. Imagine you cut cubed kilometer of wood into cubed decimeters and put them in one line to build a road which width is 1 decimeter. What is the length of this road?

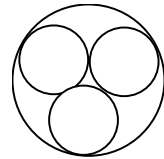
A:  $10^4$  km    B:  $10^6$  km    C:  $10^7$  km    D:  $10^9$  km    N: None of these

2. There are two types of residents on the planet called Z. Any resident of the first type has 13 arms and 13 legs, and any resident of the second type has 13 arms, 13 legs and 13 tales. All employees of The Bureau of Interplanetary Contacts of the planet Z have a total of 1313 arms, legs and tales. Find the number of employees of this Bureau if it is divisible by 13?

A: 13    B: 39    C: 45    D: 52    N: None of these

3. Three unit circles touch each other and all of them touch the bigger circle. Find the radius of this bigger circle.

A: 2    B:  $1 + \frac{2}{\sqrt{3}}$     C:  $4 - \sqrt{3}$     D: 2.5    N: None of these



4. How many 5-digit whole numbers satisfy the following condition: the product of five digits of the number is equal to 2000?

A: 10    B: 20    C: 24    D: 30    N: None of these

5. 12 chess-players have finished a tournament. Each player played exactly one time with each of the other players. A player received 2 points for each game won, 1 point for each game tied and 0 points for each game lost. Finally all players earned different number of points. Find the greatest possible number of games tied in this tournament.

A: 36    B: 40    C: 45    D: 50    N: None of these

6. Find the next to last digit of  $3^{2000}$ .

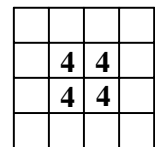
A: 0    B: 3    C: 7    D: 9    N: None of these

7. You have 6 boxes with coins that look the same. Each box contains exactly 4 coins. Coins from one of boxes weigh 1 gram each, coins from another box weigh 2 grams each, ..., coins from the last box weigh 6 grams each. The tag number (1, 2, 3, 4, 5, 6) on the top of each box should correspond to the weight of any coin from this box. You have scales without any weights and without a scale. What is the least number of attempts to weigh you should make to be sure that all tag numbers are correct?

A: 1    B: 2    C: 3    D: 4    N: None of these

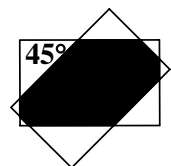
8. There are several mines in the cells of  $4 \times 4$  field (each cell contains either zero mines or one mine). Four cells are 'opened' (they have no mines), and they have numbers representing the amount of mines in neighboring cells (each cell has maximum 8 neighboring cells). In how many different ways can mines be arranged in this field?

A: 10    B: 15    C: 17    D: 20    N: None of these



9. There are 2 rectangles on the picture below. Find the ratio of the sum of 8 white triangles' perimeters to the perimeter of the shaded octagon.

A:  $\sqrt{2}$     B: 2    C:  $\sqrt{2} + 1$     D:  $\sqrt{2} + 2$     N: None of these



10. In how many different ways the numbers 1, 2, 3, 4, 5, 6 can be ordered so that a product of any two consecutive terms is an even number?

A: 100    B: 108    C: 124    D: 144    N: None of these

# Problem Sheet – 2000

## 11<sup>th</sup> – 12<sup>th</sup> Grades

1. An infinite geometrical progression contains only positive integers. The first term of this progression is a two-digit number, and the third term is equal to 100,000. Find the greatest possible value of the second term of this progression.

A: 1000      B: 2000      C: 4000      D: 5000      N: None of these

2. What number is extra in this figure?

A: 4/11      B: 1/8      C: 1/3      D:  $0.\overline{36}$       N: None of these

3. Four regular tetrahedrons that are not necessary equal were cut from the given regular tetrahedron. These cut tetrahedrons have no common points, but each of them has common vertex with the given tetrahedron. The area of the total surface of the given tetrahedron is  $100 \text{ cm}^2$ , and the sum of four cut tetrahedron' areas of the total surface is equal to  $60 \text{ cm}^2$ . Find the area of the total surface of the remaining octahedron.

A:  $40 \text{ cm}^2$       B:  $60 \text{ cm}^2$       C:  $70 \text{ cm}^2$       D:  $80 \text{ cm}^2$       N: None of these

4. How many 6-digit whole numbers satisfy the following condition: the product of six digits of the number is equal to 2000?

A: 120      B: 240      C: 360      D: 720      N: None of these

5. 12 chess-players have finished a tournament. Each player played exactly one time with each of the other players. A player received 2 points for each game won, 1 point for each game tied and 0 points for each game lost. Finally all players earned different number of points. Find the greatest possible number of games tied in this tournament.

A: 36      B: 40      C: 45      D: 50      N: None of these

6. How many ordered pairs of integers satisfy the following equation:  $x^2 + y^2 = 2000$ ?

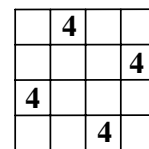
A: 4      B: 8      C: 12      D: 16      N: None of these

7. You have 6 boxes with coins that look the same. Each box contains exactly 4 coins. Coins from one of boxes weigh 1 gram each, coins from another box weigh 2 grams each, ..., coins from the last box weigh 6 grams each. The tag number (1, 2, 3, 4, 5, 6) on the top of each box should correspond to the weight of any coin from this box. You have scales without any weights and without a scale. What is the least number of attempts to weigh you should make to be sure that all tag numbers are correct?

A: 1      B: 2      C: 3      D: 4      N: None of these

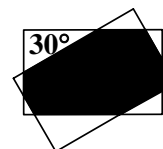
8. There are several mines in the cells of  $4 \times 4$  field (each cell contains either zero mines or one mine). Four cells are 'opened' (they have no mines), and they have numbers representing the amount of mines in neighboring cells (each cell has maximum 8 neighboring cells). In how many different ways can mines be arranged in this field?

A: 10      B: 15      C: 17      D: 20      N: None of these



9. There are 2 rectangles on the picture below. Find the ratio of the sum of 8 white triangles' perimeters to the perimeter of the shaded octagon.

A:  $\frac{5+\sqrt{3}}{3}$       B:  $\frac{3+\sqrt{3}}{2}$       C:  $2\sqrt{3}-1$       D:  $\sqrt{3}+1$       N: None of these



10. Compute the following product:  $\frac{(1^3+1) \cdot (2^3+1) \cdot (3^3+1) \cdots (100^3+1)}{(2^3-1) \cdot (3^3-1) \cdot (4^3-1) \cdots (99^3-1)}$ .

A: 999900      B: 1980000      C: 1999800      D: 2999700      N: None of these

# Answer Sheet – 2000

4<sup>th</sup> – 5<sup>th</sup> Grades

1.     A    B    C    D    N
2.     A    B    C    D    N
3.     A    B    C    D    N
4.     A    B    C    D    N
5.     A    B    C    D    N
6.     A    B    C    D    E
7.     A    B    C    D    N
8.     A    B    C    D    N
9.     A    B    C    D    N
10.    A    B    C    D    N

Total Score: 50

# Answer Sheet – 2000

6<sup>th</sup> – 7<sup>th</sup> Grades

1.     A    B    C    D    E
2.     A    B    C    D    E
3.     A    B    C    D    E
4.     A    B    C    D    E
5.     A    B    C    D    E
6.     A    B    C    D    E
7.     A    B    C    D    E
8.     A    B    C    D    E
9.     A    B    C    D    E
10.    A    B    C    D    E

Total Score: 50

# Answer Sheet – 2000

8<sup>th</sup> Grade

1.     A    B    C    D    N
2.     A    B    C    D    N
3.     A    B    C    D    N
4.     A    B    C    D    N
5.     A    B    C    D    N
6.     A    B    C    D    E
7.     A    B    C    D    N
8.     A    B    C    D    N
9.     A    B    C    D    N
10.    A    B    C    D    N

Total Score: 50

# Answer Sheet – 2000

9<sup>th</sup> – 10<sup>th</sup> Grades

1.     A    B    C    D    E
2.     A    B    C    D    E
3.     A    B    C    D    E
4.     A    B    C    D    E
5.     A    B    C    D    E
6.     A    B    C    D    E
7.     A    B    C    D    E
8.     A    B    C    D    E
9.     A    B    C    D    E
10.    A    B    C    D    E

Total Score: 50

# Answer Sheet – 2000

11<sup>th</sup> – 12<sup>th</sup> Grades

1.     A    B    C    D    N
2.     A    B    C    D    E
3.     A    B    C    D    N
4.     A    B    C    D    N
5.     A    B    C    D    N
6.     A    B    C    D    N
7.     A    B    C    D    N
8.     A    B    C    D    N
9.     A    B    C    D    N
10.    A    B    C    D    N

Total Score: 50