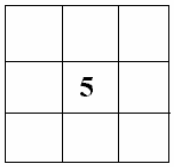
OLYMPIAD-99 PROBLEMS

4th - 5th Grades

Problem 1: Arrange the numbers 1 through 9 in 3 by 3 Magic Square, in which the sum of each of the three rows, each of the three columns, and each of the two diagonals must be equal.



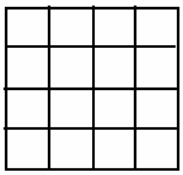
Problem 2: During a certain month three Sundays fell on even dates. What day of the week was the 25th of this month?

Problem 3: A boy has as many sisters as brothers, but each sister has only half as many sisters as brothers. How many brothers and sisters are there in the family?

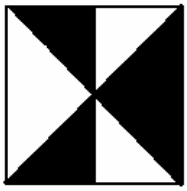
Problem 4: What is the two next numbers in this sequence?

6 9 13 18 _____ _____

Problem 5: How many squares are in this figure?



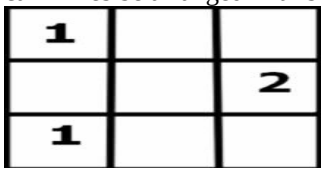
Problem 6: What fractional part of the figure is shaded?



Problem 7: People are seated at square tables. Each time a table is added additional people can sit together. Use the sequence as shown below to answer questions:
a) How many tables must be connected together in one row to seat 16 people?
b) If 18 tables are connected together in one row. How many people can be seated?

Problem 8: A building has 6 stories, each of the same height. How many minutes you will need to ascent from the first floor to the sixth if it takes 2 minutes to ascent to the third?

Problem 9: There are several mines in the cells of 3×3 field (each cell either is empty or contains exactly one mine). Tree cells are "opened" (they are empty), and they have numbers meaning the quantity of mines in neighboring cells (each cell has maximum 8 neighboring cells). In how many different ways can mines be arranged in this field?



Problem 10: Cut triangle onto 4 triangles (triangles may be of different kinds), so that any two of these triangles have no common side (but side of one triangle may be part of side of another).

OLYMPIAD-99 PROBLEMS

6th - 7th Grades

Problem 1: On what day of the week of February 7, 2099, will fall?

Problem 2: How many perfect squares are positive and less than 5,000?

Problem 3: Compute the sums:
a) 1 + 2 + 3 + 4 + ... + 97 + 98 + 99 + 100 =
b) 1 + 3 +5 + 7 + ... + 95 + 97 + 99 =
c) 2 + 4 + 6 + 8+ ....+ 94 + 96 + 98 + 100 =

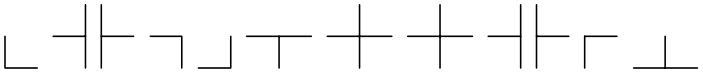
Problem 4: Sam says he knows three natural numbers *x*, *y*, and *z* that satisfy the equation $28x + 30y + 31z = 365$
What are these numbers?

Problem 5: The number 11111...11111 (100 digits) is divided by 1111111. What is the remainder?

Problem 6: George goes fishing every 18 days, mows the grass every 12 days, and deposits his pay check every 15 days. Today he did all three. How long will it be before he has another day like today?

Problem 7: The surface of a solid wooden cube is painted blue. The cube is then cut into smaller cubes, each measuring 1 cub. inch. 54 of them have only one face painted blue. What was the volume of the large cube?

Problem 8: Decode a sentence



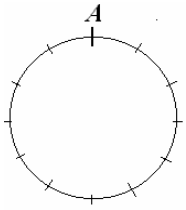
using the following key:

E	E	A
U	I	N
N	W	O
Y	R	R

Write this sentence

Problem 9:

Tom and Bom were racing bikes around a circular track. Tom rides once around in 6 minutes, and Bom can do it in 4 minutes. They start from point ***A***. In how many minutes Bom will overtake Tom at point ***A*** the first time?



Problem 10: Cut triangle onto 4 triangles (triangles may be of different kinds), so that any two of these triangles have no common side (but side of one triangle may be part of side of another).

OLYMPIAD-99 PROBLEMS

8th Grade

Problem 1: Which of the fractions: $\frac{1,998}{1,999}$ or $\frac{19,991,998}{19,991,999}$ is bigger (do not use calculator)?

Problem 2: Winnie-Pooh and Piglet are celebrating their common birthday. Every guest gives them a can of honey and a can of milk. Pooh leaves some of the cans for Piglet, and others he keeps for himself. When Pooh has eaten all of his honey, the number of full cans remained with him is equal to the total number of Piglet's cans. Piglet has 10 cans of milk. How many cans of honey have been eaten by Pooh?

Problem 3: Represent the number 651 as a sum of several natural numbers, such that the product of those numbers would be also 651?

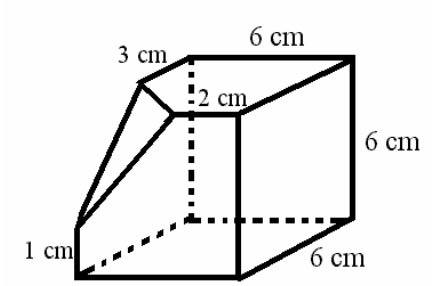
Problem 4: Divide

$$\frac{a^{128}-b^{128}}{(a+b)(a^2+b^2)(a^4+b^4)(a^8+b^8)(a^{16}+b^{16})(a^{32}+b^{32})(a^{64}+b^{64})} =$$

Problem 5: Chess tournament had a certain number of rounds. The total number of games played is 66. Determine how many players took part if every player played with all others only one time?

Problem 6: Write three digit number $\overline{xyz}$. Duplicate this number in order to get six digit number $\overline{xyzxyz}$. Prove that this six digit number divisible by 7, 11 and 13.

Problem 7: The diagram at the right shows a geometric solid formed by cutting a corner of a cube with a plane. Find the volume of this solid.



Problem 8: Eighty students went on a field trip to Seattle. 36 students visited the Aquarium, 30 went to the Woodland Park Zoo, and 33 went to the Science Center. 17 students went to both the Aquarium and the Zoo. 11 went to both the Zoo and the Science Center, and 15 went to both the Science Center and the Aquarium. 8 students went to all three. How many students did not go to any of the three places of interest?

Problem 9: Eight-graders: Andrew, Boris, Victor and George participated in NYC math competition on February 7th, 1998. The majority of participants liked problem #9. Here is what they told their math teacher the next day.

Andrew: "Boris solved this problem. Victor also solved it. And George solved this problem too".

Boris: "Andrew solved this problem. George also solved it. But Victor could not solve this problem".

Victor: "Andrew could not solve this problem. Boris also could not solve it. And George could not solve this problem too".

George: "Andrew solved this problem. Victor also solved it. But Boris could not solve this problem".

All these students made different numbers of false statements. How many students solved the problem #9, and who were they exactly?

Problem 10: Cut triangle onto 1999 triangles (triangles may be of different kinds), so that any two of these triangles have no common side (but side of one triangle may be part of side of another).

OLYMPIAD-99 PROBLEMS

9^{th} -10^{th} Grades

Problem 1: On what day of the week February 7, 2099, will fall?

Problem 2: Of 20 children in a class, 14 have brown eyes, 15 have dark hair, 17 weight more than 80 lbs, and 18 are more than 4 feet tall. How many at least children must have all four characteristics?

Problem 3: What number is in the 1999^{th} place in the sequence

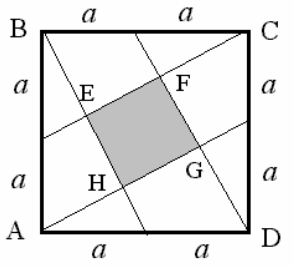
1 2 2 3 3 3 4 4 4 4 5 5 5 5 5 . . . **?**

Problem 4: Multiply out:

$$(1-x)(1+x+x^2+\cdots+x^{99}+x^{100}+x^{101}+\cdots+x^{1997}+x^{1998}+x^{1999})=$$

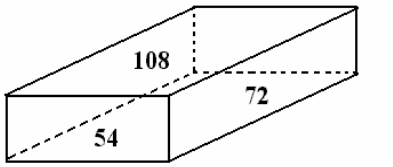
Problem 5: The positive integers 30, 72, and N have property that the product of any two of them is divisible by the third. What is the smallest possible value of N ?

Problem 6: Given a square ABCD of side 2*a* and segments drawn from each vertex to the midpoint of the second side, counting counter-clockwise, as follows. The segments intersect to form a new square HEFG. What is the ratio of the area of the shaded square to the area of the original square?



Problem 7: A watermelon composed of 99% of water, weighted 10 pounds. After a while, some of water evaporated (disappeared), leaving the watermelon composed of 98% of water. What is the new weight of watermelon?

Problem 8: A rectangle box has six surfaces. The area of its front is 54 square inches; the area of its side is 72 square inches; and the area of its top is 108 square inches. What is the volume of the box?



Problem 9: You have 6 coins in 1, 2, 3, 4, 5, 6 grams that look the same. The number (1, 2, 3, 4, 5, 6) on the top of each coin should correspond to its weight. How can you determine that all the numbers are correct, using scale balance only without any weights for just two measures?

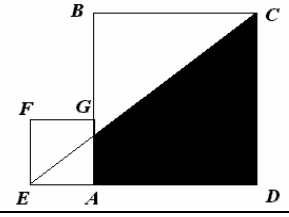
Problem 10: Cut triangle onto 1999 triangles (triangles may be of different kinds), so that any two of these triangles have no common side (but side of one triangle may be part of side of another).

OLYMPIAD-99 PROBLEMS

11^{th} - 12^{th} Grades

Problem 1: The "Cubic" firm makes wood cubes with the edge equal to 20 cm. 40 cents was spent to make one cube: the cost of wood is 30 cents, the cost of paint for the whole surface to be covered is 10 cents. The cost of labor is not counted. How much more money is needed to make cube with the edge equal to 40 cm?

Problem 2: *ABCD* and *AEFG* are squares. *AB* = 3 *AE*. Find the ratio of the shaded area to area of ABCD.



Problem 3: A watermelon composed of 99% of water, weighted 10 pounds. After a while, some of water evaporated (disappeared), leaving the watermelon composed of 98% of water. What is the new weight of watermelon?

Problem 4: Given positive numbers *x, y* and *z.*

$$\frac{y}{z}-\frac{z}{y}=a \qquad \frac{z}{x}-\frac{x}{z}=b \qquad \frac{x}{y}-\frac{y}{x}=c.$$

Find $\left(a+\sqrt{a^2+4}\right)\cdot\left(b+\sqrt{b^2+4}\right)\cdot\left(c+\sqrt{c^2+4}\right)$

Problem 5: There are 10 cows. For the first cow it takes 1 day to eat a haystack, for the second cow it takes 2 days to eat a haystack, for the third cow it takes 3 days to eat a haystack, **.....** for the tenth cow it takes 10 days to eat a haystack. Who are going to eat a haystack faster: first and second cows together or the rest cows together?

Problem 6: In how many ways the numbers 1, 2, 3, 4, 5, 6 can be ordered such that no two consecutive terms have a sum which is divisible by 2 or 3.

Problem 7: Given the number: $\underbrace{999\ldots999}_{1999\ \text{digits}}$. Find the sum of all digits of this number squared.

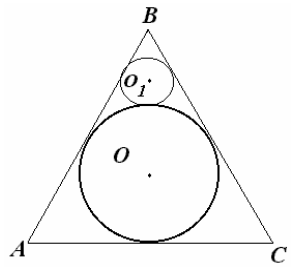
Problem 8: The sequence a_n is defined as follows:

$$a_1=1776,\quad a_2=1999,\ldots..\ a_{n+2}=\frac{a_{n+1}+1}{a_n}$$

Find a_{2002}.

Problem 9:

Circle *O* is inscribed in equilateral triangle *ABC*. *AB*=10 in . Circle O_1 touches *AB*, *BC* and circle *O*, as shown. Find the radius of the smaller circle



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