STREAMLINE SCHOOL OLYMPIAD 1998

6^{th} - 7^{th} Grades

Problem 1: In addition example

$$\begin{array}{r} ABA \\ + \\ \underline{BAB} \\ BBBC \end{array}$$

all A's denote some digit, all B's denote another digit and C denotes a third digit. What are these digits?

Problem 2: Compute the sums:

1 + 2 + 3 + 4 +... + 97 + 98 + 99 + 100 =

1 + 3 +5 + 7 +... + 95 + 97 + 99 =

2 + 4 + 6 + 8+ ...+ 94 + 96 + 98 + 100 =

Problem 3: Compute the fraction (Fibonacci number):

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+1}}}}$$

Problem 4: If you know that

$$1+\frac{1}{10}=1.1$$

$$1+\frac{1}{10}+\frac{1}{100}=1.11$$

$$1+\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}=1.111$$

find the sum (without calculation)

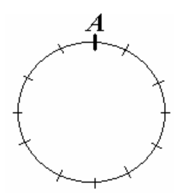
$$1+\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\frac{1}{10000}+\frac{1}{100000}+\frac{1}{1000000}=$$

Problem 5: A building has 6 floors, each the same height. How many minutes you will need to ascent from the first floor to the sixth if it takes 2 minutes to ascent to the third.

Problem 6: Three kinds of apples are mixed in a box. How many apples must you take to be sure of at least 2 apples of one kind? (Exstra: How many apples must you take to be sure of at least 3 apples of one kind?)

Problem 7: A zoo has several pony and several giraffes. They have 30 eyes and 48 legs. How many pony and giraffes are in the zoo?

Problem 8: Tob and Bom were racing bikes around a circular track. Tom rides once around in 6 minutes, and Bom can do it in 4 minutes. They start from point ***A***. In how many minutes Bom will overtake Tom at the point ***A***?



8^{th} - 9^{th} Grades

Problem 1: Which of the fraction $\frac{10001}{10002}$ and $\frac{100001}{100002}$ is bigger?	**Problem 2:** You know that $\frac{2^{1000}}{2^n} = 2^{501}$. What is n?
Problem 3: Prove that if $a+b+c=0$ then $a^3+b^3+c^3 = 3\cdot a\cdot b\cdot c$	**Problem 4:** Divide $\frac{a^{128}-b^{128}}{(a+b)(a^2+b^2)(a^4+b^4)(a^8+b^8)(a^{16}+b^{16})(a^{32}+b^{32})(a^{64}+b^{64})} =$
Problem 5: Solve the equation: $x(x+1)+(x+1)(x+2)+(x+2)(x+3)+(x+3)(x+4)+\cdots+(x+9)(x+10) = 1\cdot 2+2\cdot 3+3\cdot 4+\cdots+8\cdot 9+9\cdot 10$	**Problem 6:** Winnie-Pooh and Piglet are celebrating their common birthday. Every guest gives them a can of honey and a can of milk. Pooh leaves some of the cans for Piglet, and other he keeps for himself. When Pooh has eaten all of his honey, the amount of full cans he has left is equal to the total amount of Piglet's can. Piglet has 10 cans of milk. How many cans of honey did Pooh eat?
Problem 7: A train moving 66 feet per second meets and passed by a train moving 52 feet per second. A passenger in the first train sees the second train take 6 seconds to pass him. How long is the second train?	**Problem 8:** The garden plot is trapezoid ABCD with sides AB and CD parallel. Point E is on AD and F is on BC, with EF parallel to AB. The distance from A to E is $\frac{3}{4}$ of the distance from E to D. If segment BC is 14 feet long, how long is segment FC?
Problem 9: Find 3 numbers each of them is a square of the difference of the two others.	

10^{th} – 11^{th} Grades

Problem 1: Prove that

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+....+\frac{1}{n(n+1)}+\frac{1}{(n+1)(n+2)}=\frac{n+1}{n+2}$$

Problem 2: Determinate all three digit numbers N having the property that N is divisible by 11, and $\frac{N}{11}$ is equal to the sum of the square of the digits of N.

Problem 3: If $\frac{y}{z}-\frac{z}{y}=a$, $\frac{z}{x}-\frac{x}{z}=b$, $\frac{x}{y}-\frac{y}{x}=c$, prove that

$$\left(a+\sqrt{a^2+4}\right)\cdot\left(b+\sqrt{b^2+4}\right)\cdot\left(c+\sqrt{c^2+4}\right)=8$$

Problem 4: In a library system with 6 branches, 60 workers are employed. If no library has fewer than 7 workers and no more than 11, what is the minimum number of workers in any 2 of the branches?

Problem 5: If a man walks S miles at rate R_1 miles per hour, and then the same distance at rate R_2 miles per hours, what is his average rate for the entire trip.

Problem 6: Find the value of the expression

$$\left(1+3^2+5^2+7^2+9^2+11^2+...+(2n-1)^2+...+195^2+197^2+199^2\right)-$$
$$\left(2^2+4^2+6^2+8^2+10^2+...++(2n)^2+...+196^2+198^2+200^2\right)=$$

Problem 7: Compute the sum: $1+\frac{1}{2}+$

$$+1+\frac{1}{2}+\frac{1}{2^2}+$$
$$+1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+$$
$$+1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+$$
$$+..+$$
$$+1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\cdots+\frac{1}{2^9}+\frac{1}{2^{10}}$$

Problem 8: Find the number a and the digit symbolized by c in the following equality
:

$$9\cdot(230+a)^2=492c04$$

Problem 9: There are 3 boxes with balls inside them. They are labeled "two whites", "two blacks" and "black and white" to show the contents of the boxes. Someone changes the labels to make them all false. Find the contents of each box by removing only one of the balls.

Problem 10: In the figure, determine the area of the shaded octagon. ***ABCD*** is a square, ***K, L, M, N*** are midpoints, AB = 4 ft.