Streamline School
Phone: (718) 745-2370
E-mail: varbo@earthlink.net
Website: http://www.streamlineschool.org

Advanced Math Academy
Phone: (201) 321-9756
E-mail: Advanced_Math_Academy@yahoo.com

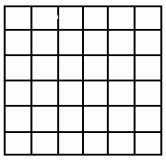
Problem Sheet – 2006

4^{th} – 5^{th} Grades, Second Round

1. All even natural (i.e. positive whole) numbers from 2 till 2006 are recorded in order: 246810 … 20042006. How many digits are there in the resulting number?

2. A set of three mutually distinct (all three are different) numbers is "beautiful" if for every two of them one of these two numbers equals to another one either multiplied or increased by 2. Find the number of "beautiful" sets of three natural (i.e. positive whole) numbers. "Beautiful" sets that have the same numbers (even if listed in different order) should be considered the same.

3. A square 6×6 made from 36 unit cells is cut along the cell sides into several mutually distinct (all are different) rectangles. Find the greatest possible number of rectangles in such "decomposition". Rectangles like 1×2 and 2×1 should be considered the same.



4. In the expression 1 + 2 + 3 + 4 + … + 99 + 100 it is allowed to replace several "+" signs with "–" signs. An expression is selected if its value is as close to 2006 as possible. Find the value of the selected expression.

5. Each side of a square is divided into 3 equal segments using 12 points (including 4 vertices of the square). How many triangles have all their vertices among these 12 points?

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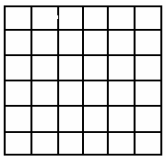
Problem Sheet – 2006

6^{th} – 7^{th} Grades, Second Round

1. All odd natural (i.e. positive whole) numbers from 1 till 2007 are recorded in order: 13579 … 20052007. How many digits "one" (1) are there in the resulting number?

2. A set of three mutually distinct (all three are different) numbers is "beautiful" if for every two of them one of these two numbers equals to another one either multiplied or increased by 2. Find the number of "beautiful" sets of three integer numbers. "Beautiful" sets that have the same numbers (even if listed in different order) should be considered the same.

3. A square 6×6 made from 36 unit cells is cut along the cell sides into 8 mutually distinct (all are different) rectangles. For any such "decomposition" the rectangle containing the most number of cells is selected. Find the greatest possible number of cells in the selected rectangle. Rectangles like 1×2 and 2×1 should be considered the same.



4. In the expression 1 + 2 + 3 + 4 + … + 2005 + 2006 it is allowed to replace several "+" signs with "–" signs. An expression is selected if its value is as close to 2006 as possible. Find the value of the selected expression.

5. Each side of a square is divided into 4 equal segments using 16 points (including 4 vertices of the square). How many right triangles have all their vertices among these 16 points?

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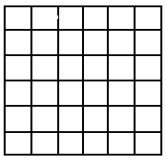
Problem Sheet – 2006

8th Grade, Second Round

1. All natural numbers from 1 till 2006 are recorded in order: 1234 … 20052006. How many pairs of consecutive digits "one" (1) are there in the resulting number?

2. A set of three mutually distinct (all three are different) numbers is "beautiful" if for every two of them one of these two numbers equals to another one either multiplied or increased by 2. Find the number of "beautiful" sets of three positive numbers. "Beautiful" sets that have the same numbers (even if listed in different order) should be considered the same.

3. A square 6×6 made from 36 unit cells is cut along the cell sides into 8 mutually distinct (all are different) rectangles. For any such "decomposition" the rectangle containing the most number of cells is selected. Find the least possible number of cells in the selected rectangle. Rectangles like 1×2 and 2×1 should be considered the same.



4. In the expression 1 + 2 + 3 + 4 + … + 2005 + 2006 the only following modifications are allowed:

a) to replace exactly one "+" sign with "–" sign;
b) to put "(" immediately after this "–" sign;
c) to put ")" at the very end, after 2006.

An expression is selected if its value is as close to 2006 as possible. Find the value of the selected expression.

5. Each side of a square is divided into 4 equal segments using 16 points (including 4 vertices of the square). How many right triangles have all their vertices among these 16 points?

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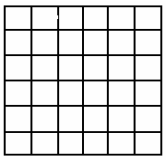
Problem Sheet – 2006

9^{th} – 10^{th} Grades, Second Round

1. All natural numbers from 1 till 2006 are recorded in order as Roman numerals: IIIIIIIVV … MMVMMVI. How many Roman digits "one" (I) are there in the resulting record?

2. A set of three mutually distinct (all three are different) numbers is "beautiful" if for every two of them one of these two numbers equals to another one either multiplied or increased by 2. Find the number of "beautiful" sets of three rational numbers. "Beautiful" sets that have the same numbers (even if listed in different order) should be considered the same.

3. A square 6×6 made from 36 unit cells is cut along the cell sides into the greatest possible number of mutually distinct (all are different) rectangles. For any such "decomposition" the rectangle containing the most number of cells is selected. Find the greatest possible number of cells in the selected rectangle. Rectangles like 1×2 and 2×1 should be considered the same.



4. In the expression 1 + 2 + 3 + 4 + … + 2005 + 2006 the only following modifications are allowed:

a) to replace exactly one "+" sign with "–" sign;
b) to put "(" immediately after this "–" sign;
c) to put ")" at the very end, after 2006.

An expression is selected if its value is as close to 2006 as possible. Find the value of the selected expression.

5. In square *ABCD P* is the midpoint of the side *BC*. Points *Q* and *R* are selected on the side *AD* in such way that $4AQ = 4DR = AD$. Calculate the sum of the measures (in degrees) of angles *ACQ*, *BRP*, and *ABQ*.

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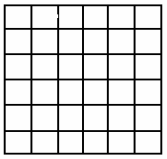
Problem Sheet – 2006

11^{th} – 12^{th} Grades, Second Round

1. All natural numbers from 1 till 2006 are recorded in order as Roman numerals: IIIIIIIVV … MMVMMVI. How many Roman digits are there in the resulting record?

2. A set of three mutually distinct (all three are different) numbers is "beautiful" if for every two of them one of these two numbers equals to another one either multiplied or increased by 2. Find the number of "beautiful" sets of three real numbers. "Beautiful" sets that have the same numbers (even if listed in different order) should be considered the same.

3. A square 6×6 made from 36 unit cells is cut along the cell sides into the greatest possible number of mutually distinct (all are different) rectangles. For any such "decomposition" the rectangle containing the most number of cells is selected. Find the least possible number of cells in the selected rectangle. Rectangles like 1×2 and 2×1 should be considered the same.



4. In the expression 1 + 2 + 3 + 4 + … + 2005 + 2006 the only following modifications are allowed:

a) to replace exactly one "+" sign with "–" sign;

b) to put "(" immediately after this "–" sign;

c) to put ")" immediately after some number on the right of the "–" sign (but not necessary after the number next to the "–" sign).

An expression is selected if its value is as close to 2006 as possible. Find the value of the selected expression.

5. In square *ABCD* point *P* is selected on the side *AB*, points *Q* and *R* are selected on the side *BC*, and point *S* – on the side *AD* in such way that $3BP = 3BQ = 3CR = 3DS = AD$. Calculate by how many degrees is the sum of the measures of angles *BRP*, *BSQ*, and *SPD* greater than the measure of angle *RPC*.