Streamline School
Phone: (718) 745-2370
E-mail: varbo@earthlink.net
Website: http://www.streamlineschool.org

Advanced Math Academy
Phone: (201) 321-9756
E-mail: Advanced_Math_Academy@yahoo.com

Problem Sheet – 2008

Welcome to the Tenth Annual Mathematics Olympiad of Streamline School & Advanced Math Academy!

The competition consists of **8 problems**. In **two hours** you have to provide numeric answers on the Answer Sheet. Make your answers clear and readable, but do not include any word, just numbers. All answers should be provided in the simplest form. Be careful not to make any stray marks on the Answer Sheet. If you change an answer, erase your first answer completely. Or you can ask for a new blank Answer Sheet. If a problem does have multiple answers you have to list ALL of them. If your Answer Sheet misses any correct answer or shows at least one incorrect answer in response to a question, that item will be scored as incorrect. Do not fold or tear the Answer Sheet.

You also have several Explanation Sheets to show your work and explain your answers. Put all your calculations and explanations on these sheets. Do ask assistants if you need additional Explanation Sheets.

5 ✓		WRONG
5 ~~8~~		WRONG
5	4	WRONG
10/2		WRONG
Answer is 5		WRONG
5		RIGHT

Each problem is worth 5 points. **Correct answer = 2 points, incorrect (or incorrectly marked) = 0 points. You can earn up to 3 additional points for completeness and correctness of your explanation, but only if your answer is correct.**

There is no penalty for guessing. If you are not sure about the answer, put your best guess. If your guess is correct, you will get 2 points for the answer and 0 points for no explanation or an explanation such as: 'I made a guess.' If you can, try to explain your guesses to get up to 3 more points for every problem.

Do not spend too much time on any one question for the first time. Answer and explain each question as best as you can, or skip it and keep going. If you have time at the end of the competition, you can go back.

You cannot use calculators, mobile phones, or reference books to solve Olympiad problems. And of course, you cannot ask other students for help. **Do ask assistants if you cannot understand a problem, have some questions about a problem (not a solution), or need to translate it into Russian. Solutions/explanations could be written in any language.**

Good luck!

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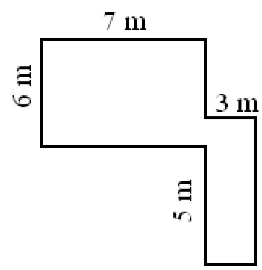
Problem Sheet – 2008

4^{th} – 5^{th} Grades

1. When the six-digit number 3456*N*7 is divided by 8, the remainder is 5. List all possible values of the digit *N*.

2. How many positive factors does number 72 have?

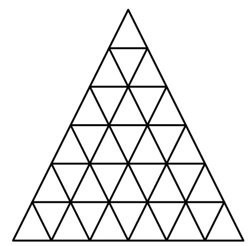
3. If 1 Tom is equal to 7 Fan, 2 Harry is equal to 1 Husk, and 14 Tom is equal to 1 Husk, then how many Fan make up a Harry?



4. What is the perimeter (in meters) of the figure shown? All angles are right angles.

5. Bay Street has between 2 and 15 houses, numbered 1, 2, 3, and so on. Mr. Sullivan lives in one of the houses. The sum of all the house numbers less than his equals the sum of all the house numbers greater than his. How many houses are there on Bay Street?

6. Let's define an 8-digit positive whole number *abcdefgh* (*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h* – its digits, *a* > 0) as "*beautiful*" if numbers *aceg* and *bdfh* are the same (for example, 22000088.) Find the number of "*beautiful*" numbers.



7. How many triangles are in this figure? Count all triangles you can see without drawing additional lines.

8. There are 6 coins that look the same, each of them in its own clear plastic bag (bags are labeled using letters *A*, *B*, *C*, *D*, *E*, *F* and have the same weight.) 3 of these coins are genuine, having the same weight, and 3 others are fake, also having the same weight, but a fake coin is lighter than a genuine one. There is also a correctly functioning balance without any reference weights and without a scale. What is the least number of weigh attempts an expert should make to surely identify all genuine coins? The expert does not have any additional coins.

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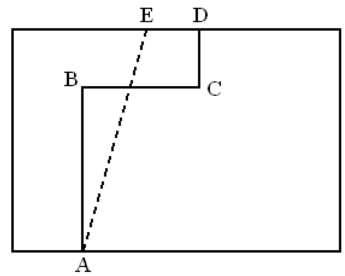
Problem Sheet – 2008

6th – 7th Grades

1. My watch *bips* every minute. Jim's watch *bops* every 62 seconds. If our *bip* and *bop* coincide at 12:00 pm, what is the next time our *bip* and *bop* coincide?

2. Find the sum of the first 100 terms of the series: $\underbrace{1+2+5+6+9+10+13+14+\ldots+197+198}_{100\text{ numbers}}$.

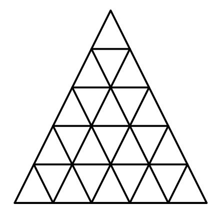
3. Which number is greater: 2008^{100} or $2007^{100}+2007^{99}$?



4. Two pieces of rectangular land are separated by the line *ABCD*, as shown on the diagram. *AB* = 30 m, *BC* = 24 m, and *CD* = 10 m. *AB*, *BC*, and *CD* are parallel to the sides of the rectangle. We want to straighten the border line by the straight line *AE*, so that the areas of both land pieces would not change. What is the distance *ED* (in meters)?

5. Based on a survey conducted in a group of 13-year olds, it turned out that they watch TV for an average of 50 minutes every day. The average time the boys in this group watch TV was 45 minutes per day, and the average time the girls in this group watch TV was 65 minutes per day. What is the ratio of the numbers of the boys and the girls in this group?

6. Let's define an 8-digit positive whole number *abcdefgh* (*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h* – its digits, $a > 0$) as "*beautiful*" if at least two of its neighboring digits are the same (for example, 20080316 or 20000000.) Find the number of "*beautiful*" numbers.



7. How many quadrilaterals are in this figure? Count all quadrilaterals you can see without drawing additional lines.

8. There are 6 coins that look the same, each of them in its own clear plastic bag (bags are labeled using letters *A*, *B*, *C*, *D*, *E*, *F* and have the same weight.) At least 4 of these coins are genuine, having the same weight, and the others (may be none) are fake, also having the same weight, but a fake coin is lighter than a genuine one. There is also a correctly functioning balance without any reference weights and without a scale. What is the least number of weigh attempts an expert should make to surely identify all genuine coins? The expert does not have any additional coins.

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Problem Sheet – 2008

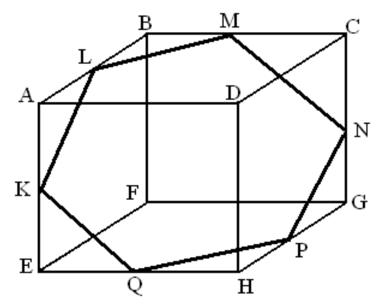
8^{th} Grade

1. A palindrome is a whole number (first digit cannot be 0) that reads the same front to back or back to front (for example, 13231.) Compute the number of palindromes between 10 and 1000.

2. Which number is greater: 2008^{100} or $2007^{100} + 2007^{99}$?

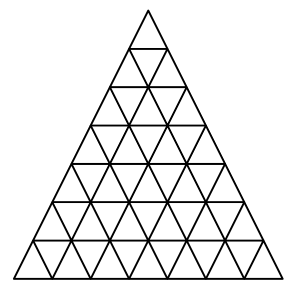
3. If p represents a positive prime number then what is the smallest p such that $p^{13} + 5p^{12}$ is a perfect cube?

4. A confused bank teller transposed the dollars and cents when he cashed a check for Ms. Smith, giving her dollars instead of cents and cents instead of dollars. After buying a book for 9 dollars and 90 cents, Ms. Smith noticed that she had left exactly the amount of the original check. What was the amount of the check? Ms. Smith had no money before cashing the check.



5. *ABCDEFGH* is a cube with side length 1. *K*, *L*, *M*, *N*, *P*, *Q* are the midpoints of *AE*, *AB*, *BC*, *CG*, *GH*, *HE* respectively. Compute the area of the regular hexagon *KLMNPQ*.

6. For every 8-digit positive whole number *abcdefgh* (*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h* – its digits, *a* > 0) let's define its "*beauty*" as the value of the expression *a* – *b* + *c* – *d* + *e* – *f* + *g* – *h* (for example, number 20080316 has a "*beauty*" -14.) Find an average "*beauty*" of all 8-digit positive whole numbers.



7. How many parallelograms are in this figure? Count all parallelograms you can see without drawing additional lines. All lines shown are parallel to the sides of the big triangle.

8. There are 10 coins that look the same, each of them in its own clear plastic bag (bags are labeled using letters *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J* and have the same weight.) 5 of these coins are genuine, having the same weight, and 5 others are fake, also having the same weight, but a fake coin is lighter than a genuine one. There is also a correctly functioning balance without any reference weights and without a scale. An attorney knows (from his client) which 5 coins are fake ones. But he cannot identify (directly or indirectly) any of fake coins to a jury, otherwise his client would be punished. Therefore the attorney cannot make weigh attempts that would lead to logical identification of at least one of the fake coins. To defend the client, the attorney has to split (with an explanation that satisfies the jury) the given 10 coins into 5 pairs having one genuine and one fake coins each. What is the least number of weigh attempts the attorney should make to surely reach this goal? The attorney does not have any additional coins.

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Problem Sheet – 2008
9^{th} – 10^{th} Grades

1. How many consecutive zeros are there at the end of the number $\frac{30!}{4 \times 5!}$?

2. The numbers $\sqrt[8]{3}$, $\sqrt[4]{3}$, and x are respectively first, second and third terms of a geometric sequence. If x is in the form 3^y, find y.

3. The increasing sequence 2, 3, 5, 6, 7, 10, … consists of all natural numbers which are neither perfect squares not perfect cubes. Find the 75^{th} term of this sequence.

4. Let ABC be a triangle. Suppose that $AB = x + 4$, $BC = x + 7$, and $AC = 4x$. For angle CAB to be the only largest angle of the triangle, x must satisfy condition $m < x < n$. Find the sum of the least possible value of n and the most possible value of m.

5. Jessie has some 7¢, some 13¢, and some 37¢ stamps. She has the same number of two kinds of these stamps and a different number of the third kind. The total value of all her stamps is $2.49. How many of each kind of stamps does she have?

6. Let's define an 8-digit positive whole number *abcdefgh* (*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h* – its digits, $a > 0$) as "*beautiful*" if number *aceg* is greater than number *bdfh* (for example, 20080316.) Find the number of "*beautiful*" numbers.

7. In a convex hexagon *ABCDEF* quadrilaterals *ABDE*, *BCEF*, and *CDFA* are parallelograms, *K*, *L*, and *M* are the midpoints of *AB*, *CD*, and *EF* respectively. Find the ratio of the area of the triangle *KLM* to the area of the hexagon *ABCDEF*.

8. There are 10 coins that look the same, each of them in its own clear plastic bag (bags are labeled using letters *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J* and have the same weight.) 5 of these coins are genuine, having the same weight, and 5 others are fake, also having the same weight, but a fake coin is lighter than a genuine one. There is also a correctly functioning balance without any reference weights and without a scale. An attorney knows (from his client) which 5 coins are fake ones. But he cannot identify (directly or indirectly) any of fake coins to a jury, otherwise his client would be punished. Therefore the attorney cannot make weigh attempts that would lead to logical identification of at least one of the fake coins. To defend the client, the attorney has to identify (with an explanation that satisfies the jury) any genuine coin obeying the following restriction: each time he has to place exactly two coins on each (left and right) balance pans. What is the least number of weigh attempts the attorney should make to surely reach this goal? The attorney does not have any additional coins.

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Problem Sheet – 2008

11^{th} – 12^{th} Grades

1. Find the smallest positive whole number that ends with 17, is divisible by 17, and the sum of its digits is 17.

2. The numbers $\sqrt[8]{3}$, $\sqrt[4]{3}$, and x are respectively first, second and third terms of a geometric sequence. If x is in the form 3^y, find y.

3. What is the area of the region in the Cartesian Plane that is defined by the following inequality: $|x|+|y|+|x+y| \le 2$?

4. How many consecutive zeros are there at the end of the number $9^{999}+1$?

5. What is the coefficient of x^{99} in the expansion of $(x-1)(x-2)\ldots(x-100)$?

6. Let's define an 8-digit positive whole number *abcdefgh* (*a*, *b*, *c*, *d*, *e*, *f*, *g*, *h* – its digits, $a > 0$) as "*beautiful*" if $a - b + c - d + e - f + g - h > 0$ (for example, 20089123.) Find the number of "*beautiful*" numbers.

7. In a convex hexagon *ABCDEF* for each pair of its opposite sides one of these sides (let's call it "*long*") is parallel to and 50% longer than another one (let's call it "*short*".) *K*, *L*, and *M* are the midpoints of the hexagon's "*short*" sides. Find the ratio of the area of the triangle *KLM* to the area of the hexagon *ABCDEF*.

8. There are 10 coins that look the same, each of them in its own clear plastic bag (bags are labeled using letters *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J* and have the same weight.) 5 of these coins are genuine, having the same weight, and 5 others are fake, also having the same weight, but a fake coin is lighter than a genuine one. There is also a correctly functioning balance without any reference weights and without a scale. An attorney knows (from his client) which 5 coins are fake ones. But he cannot identify (directly or indirectly) any of fake coins to a jury, otherwise his client would be punished. Therefore the attorney cannot make weigh attempts that would lead to logical identification of at least one of the fake coins. To defend the client, the attorney has to identify (with an explanation that satisfies the jury) some genuine coins (the more the better) using just 3 weigh attempts and obeying the following restriction: each time he has to place exactly two coins on each (left and right) balance pans. What is the most number of genuine coins the attorney could identify to the jury? The attorney does not have any additional coins.